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Research Article

Numerical Solution of Continuation Problem for 3D Steady-State Diffusion in Cylindrically Layered Medium

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This work is based on the application of Fourier and quasi-solution methods for solving the continuation inverse problem for 3D steady-state diffusion model inside a cylindrical layered medium. The diffusion coefficient is supposed to be a piecewise constant function, Cauchy data are given on the outer boundary of the cylinder, and we seek to recover the temperature at the inner boundary of the cylinder. Numerical experiments are investigated and show the capacity of proposed method only for smooth boundary condition. Under the suitable choice of regularization parameters we recover the distribution of temperature on the inner boundary with satisfactory quality for noisy data.

1. Introduction

The Cauchy problem for Laplace's equation arises from many branches of science and engineering such as medicine [1, 2], geophysics [3], and nondestructive testing [4]. It is so important to reconstruct data on the boundary because the object of interest of the boundary is not available for measurement in practice. As an additional information all possible measurements are used on the available part of the boundary. Numerical computations become very difficult if there is no a priory information on the solution. A small error in the data deduces large error in the solution. Usually the regularization technique is required to find a stable approximate solution. In order to obtain a stable numerical solution for these kinds of ill-posed problems many regularization methods have been proposed [5–21]. Application of spectral methods to the Cauchy problem for Laplace's equation was used for the first time by Lavrent'ev in [5]. A lot of regularization methods are implemented for a Cauchy problem of Laplace's equation by many authors in [6–9]. A new regularization method based on a finite dimensional subspace generated from fundamental solution for solving a Cauchy problem of Laplace's equation in a simply connected bounded domain is proposed in [6]. A spectral regularization method for a Cauchy problem to Laplace's equation in a rectangle is implemented by authors of [7]. A spectral method together with choice of regularization parameter is presented and error estimate is obtained in [8]. A semidiscrete difference scheme together with a choice of regularization parameter is presented and error estimate is obtained for the Cauchy problem of Laplace's equation in [9].

The application of the method of fundamental solution with invariant condition to determine an unknown portion of the boundary in the domain from the Cauchy data connected with the Laplace equation is studied in [10].

A Cauchy problem of Laplace's equation in a multiconnected domain, that is, determining the temperature or heat flux on the inner inaccessible boundary from Cauchy data on the outer accessible boundary, is considered by authors in [11]. The method of fundamental solutions is applied to solve considered problem. They study the application of the method of fundamental solutions to solve the Cauchy problem of Laplace's equation based on Tikhonov regularization method with L-curve choice strategy for choosing the regularization parameter.

To obtain stable numerical solution for a Cauchy problem to the Laplace equation the authors of paper [12] proposed a mollification method. The idea of this method is very simple: