Pseudopotentials of the particles interactions in complex plasmas

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(Received 16 June 2011; accepted 13 September 2011; published online 18 October 2011)

This article discusses the effective interaction potentials in a complex dusty plasma. The interaction of electrons with atoms and the interaction between dusty particles are studied by the method of the dielectric response function. In the effective interaction, potential between electron and atom the quantum effects of diffraction were taken into account. On the curve of the interaction potential between dust particles under certain conditions the oscillations can be observed. © *2011 American Institute of Physics*. [doi:10.1063/1.3646924]

I. INTRODUCTION

Plasma is the most common state of matter in the universe. It exists in a wide range of temperatures and densities. The plasma may be observed not only in gases, but also in solids. Charged particles interact via Coulomb forces which are long range. Consequently, the physical properties of the plasma significantly differ from those of ordinary neutral gases. Collective effects that appear in the plasma due to the long-range nature of Coulomb forces can be studied using effective interaction potentials. Effective interaction potential of particles which takes into account many particle correlation effects can be found by two methods. The first is a method of hierarchy which leads to a generalized Poisson equation and the second one is the method of dielectric functions.

At present active investigations of the complex plasma are performed in the different laboratories. Study of complex plasmas for applications became especially important after finding of dust structures in the plants of the plasma spraying, because such structures have a negative impact on the quality of deposited samples. More over it was known that the complex plasma is widely distributed in the space.

Quantum effects in the complex low pressure gas-discharge plasma do not play a big role, but they are important in space plasma. Therefore, the study in the classical approximation is as urgent as in the semiclassical one. One or another approach should be reflected in the interaction potential of particles. For classical plasma screening at large distances is most important effect in the interaction of charged particles, whereas in the semiclassical approximation the diffraction effects at small interparticle distances should be taken into account as accurately as the screening. The interaction micro-potentials of the charged particles taking into account the effects of diffraction are presented in Refs. 1 and 2.

Interaction of electron and atom is connected with the polarization of atom under the influence of an electric field of the electron and quantum effects, including the symmetry of the wave function. In Refs. 3 and 4, the screening potential of Buckingham is used as an effective potential. Interactions of charged particles with atoms were considered in detail in Refs. 5 and 6.

Different mechanisms of interaction between dusty particles in plasma are considered in Ref. 7, where it is shown that the dominant mechanism of interaction depends on the parameters of the surrounding plasma. The mechanism of the dipole interaction between the dust particles is studied and an estimation of it is given in Ref. 8.

In the most experiments on the complex plasma dust component is strongly coupled, and the plasma environment (buffer plasma), in contrast, is weakly non-ideal. The latter justifies using of the theory of linear dielectric response in order to find potential interactions. Interaction potentials of electrons, ions, and atoms in a wide range of temperature and concentration in classical and semiclassical approximations are considered in Refs. 9–12.

II. DIMENSIONLESS PARAMETERS

We consider the complex plasma, consisting of electrons, ions, atoms, and dust particles. The average distance between dust particles is determined by their concentration

$$a = \left(\frac{3}{4\pi n}\right)^{1/3}.$$

Coupling parameter is defined as the ratio of interaction energy to the energy of thermal motion

$$\Gamma = (Ze)^2/(ak_BT),$$

where Z is the charge number, T is plasma temperature, and k_B is the Boltzmann constant.

Density parameter: $r_S = a/a_B$, here $a_B = \hbar^2/(me^2)$ is the Bohr radius.

The degree of degeneracy is determined by the Fermi energy

$$\Theta = \frac{k_BT}{E_F} = 2\left(\frac{4}{9\pi}\right)^{2/3} Z^{5/3}\frac{r_s}{\Gamma},$$

where E_F is the Fermi energy of electrons.

III. PSEUDOPOTENTIAL OF THE ELECTRON-ATOM INTERACTION

In a neutral atom, interacting with a free electron outer electron cloud is polarized. At long distances, the interaction

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potential of the isolated atom with electron (see Ref. 13) is given by

$$\varphi_0 = -\frac{e^2\alpha}{2r^4}. \tag{1}$$

Singularity at small distance is removed for the finite size of the atom. In this case, the cutoff radius is introduced

$$\varphi_0 = -\frac{e^2\alpha}{2(r^2+r_B^2)^2}, \tag{2}$$

where $r_B = \sqrt[4]{\frac{\alpha a_B}{2}}$ is cutoff radius.

It is known that in the plasma, any electrostatic interaction is screened at large distances. A semi-empirical potential taking into account the screening effect is presented in Ref. 14

$$\Phi(r) = -\frac{e^2\alpha}{2(r^2+r_B^2)^2}\exp\left(-\frac{2r}{r_D}\right)\left(1+\frac{r}{r_D}\right)^2, \tag{3}$$

where $r_D = \sqrt{k_B T/4\pi n e^2}$ is the Debye length. In Ref. 9, the effective interaction potential with taking into account the screening and diffraction effects was obtained

$$\Phi(r) = -\frac{e^2\alpha}{2r^4(1-4\lambdabar^2/r_D^2)}\left(e^{-Br}(1+Br)-e^{-Ar}(1+Ar)\right)^2, \tag{4}$$

here, $\lambdabar = \hbar/\sqrt{2\pi m_e k_B T}$ is the thermal electron de Broglie wavelength because $m_e << m_a$.

$$A^2 = \frac{1}{2\lambdabar^2}\left(1+\sqrt{1-4\lambdabar^2/r_D^2}\right),$$

$$B^2 = \frac{1}{2\lambdabar^2}\left(1-\sqrt{1-4\lambdabar^2/r_D^2}\right).$$

We try to find an interaction pseudopotential in the random phase approximation on the basis of micropotential (1). From linear response theory, it is known that Fourier transform of the pseudopotential is obtained from the relation

$$\tilde{\Phi}(q) = \frac{\tilde{\varphi}(q)}{\varepsilon(q)}, \tag{5}$$

where $\varepsilon(q)$ is the Fourier transform of dielectric functiona, $\tilde{\varphi}(q)$ is the Fourier transform of the micropotential. The Fourier transform is calculated as

$$\tilde{\varphi}(q) = \frac{4\pi}{q}\int_0^\infty r\varphi(r)\sin(qr)dr. \tag{6}$$

$\varepsilon(q)$ in the random phase approximation can be expressed as

$$\varepsilon(q) = 1 + \sum_\alpha n_\alpha \tilde{\Phi}(q)/k_B T. \tag{7}$$

The Fourier transform of micropotential (2) founded by the formula (6) is

$$\tilde{\varphi}(q) = -\frac{\pi^2 e^2\alpha}{2r_B}\exp(-qr_B). \tag{8}$$

In the case of the Coulomb interaction in the classical limit, the dielectric function is

$$\varepsilon(q) = \frac{q^2+k_D^2}{q^2}, \tag{9}$$

where $k_D^2 = 1/r_D^2$. From Eq. (5) using Eqs. (8) and (9), one can obtain

$$\Phi(r) = -\frac{e^2\alpha}{2(r^2+r_B^2)^2} + \frac{e^2\alpha}{4r_B r_D^2}\frac{iG(r)}{r},$$

here

$$G(r) = e^{-ir_B/r_D}(Ei((r+ir_B)/r_D) - Ei(-(r-ir_B)/r_D)) + e^{ir_B/r_D}(Ei(-(r+ir_B)/r_D) - Ei((r-ir_B)/r_D)),$$

$Ei(z) = -\Gamma(0, ze^{-\pi i}) = -\int_z^{-\infty}\frac{e^t}{t}dt$ is the exponential integral function.

One can expand special functions into a series and retain only terms up to third order and then obtain

$$\Phi(r) = -\frac{e^2\alpha}{2(r^2+r_B^2)^2} + \frac{e^2\alpha\cos(r_B/r_D)}{4r_B r_D^2}\frac{[\pi/2 - arctg(r_B/r)]}{r}, \tag{10}$$

where $r_D > r_B$. First term in Eq. (10) is equal to the micropotential (2), while the second term describes the screening at large distances, and tends to zero at short ones. When the potential of Deutsch $\phi = e^2\exp[-\lambdabar/r]/r$ is selected as micropotential for electrons interaction, one can obtain the dielectric function in the semiclassical limit

$$\varepsilon(q) = \frac{q^2(q^2+1/\lambdabar^2)+k_D^2/\lambdabar^2}{q^2(q^2+1/\lambdabar^2)}. \tag{11}$$

From formula (5), using Eqs. (8) and (11) and performing calculations similar to the definition (10) it is possible to find

$$\Phi(r) = -\frac{e^2\alpha}{2(r^2+r_B^2)^2} + \frac{e^2\alpha(\cos(r_B C_2)-\cos(r_B C_1))}{4r_B r_D^2\sqrt{1-4\lambdabar^2/r_D^2}}\frac{[\pi/2 - arctg(r_B/r)]}{r}, \tag{12}$$

here

$$C_1^2 = \left(1+\sqrt{1-4\lambdabar^2/r_D^2}\right)/(2\lambdabar^2),$$

$$C_2^2 = \left(1-\sqrt{1-4\lambdabar^2/r_D^2}\right)/(2\lambdabar^2), \tag{13}$$

where $r_D > r_B$. The second term in Eq. (12) describes the screening effects at large distances and diffraction at small.

Fig. 1 shows micropotential (2) and the effective interacting potential (10) for the electron and hydrogen atom

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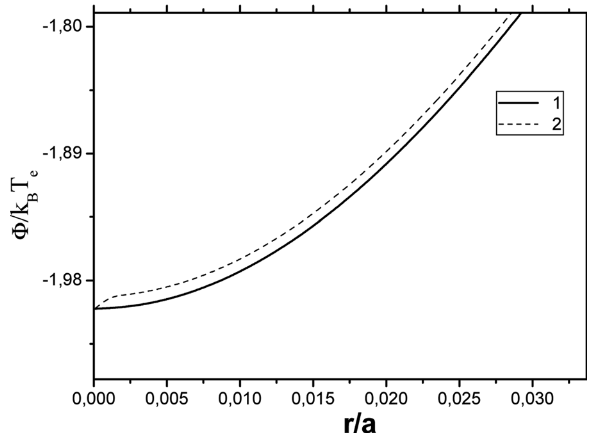

FIG. 1. Interaction potentials between electron and atom. Curve 1 presents the micropotential (2) and curve 2 presents the effective potential (10). $\Gamma = 0.2$ and $r_s = 10$.

($\alpha = 4.5a_B^3$) at $\Gamma = 0.2, r_s = 10$. It is evident that the potential (10) lies above the potential (2) what is due to the screening effect. Fig. 2 shows plots for the potentials (2), (10), and (12) at $\Gamma = 1, r_S = 10$. At large distances, potentials (10) and (12) coincide, and at the small ones, curve (12) lies above the curve (10) due to the effects of diffraction. Potentials (2), (10), and (12) coincide again when $r = 0$. It is approximately assumed that α is constant at any distance.

IV. THE POTENTIAL INTERACTION BETWEEN CHARGED PARTICLES WITH A DIPOLE MOMENT

Lets consider a system of two dust particles which have the same charges Ze. It is known that in plasma dust particles can have dipole momentums (see Ref. 8). The first dust particle is situated in the field of second one. The mutual distance between dust particles is larger than their sizes. Then, the total potential energy of the system can be expanded in series (see Ref. 15)

$$U = U_0 + U_1 + \dots . \tag{14}$$

Here, $U_0 = \varphi_0 \sum eZ = \frac{e^2Z^2}{r} - \frac{\vec{d}_1\vec{n}}{r^2}eZ$,

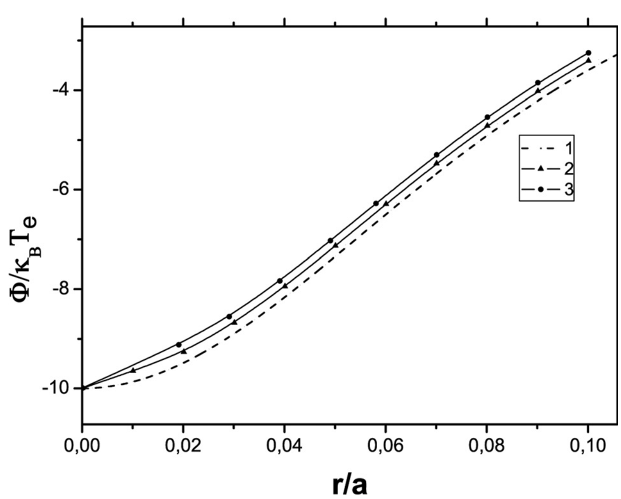

FIG. 2. Interaction potentials between electron and atom. Curve 1 presents the micropotential (2), curve 2 presents the effective potential (10), and curve 3 presents the effective potential (12). $\Gamma = 1$ and $r_s = 10$.

$$U_1 = -\vec{d}_2\vec{E}_0 = \frac{eZ}{r^2}\vec{n}\vec{d}_2 + \frac{\vec{d}_1\vec{d}_2 - 3\left(\vec{n}\vec{d}_1\right)\vec{n}\vec{d}_2}{r^3}, \tag{15}$$

$\vec{d}_1, \vec{d}_2$ are the dust particles dipole momentums, and at the condition $\frac{2a_D}{r} << 1$ (a_D is the size of the dust particle), the following expression is obtained

$$U \approx \frac{e^2Z^2}{r} + \frac{eZ}{r^2}\left(\vec{d}_2 - \vec{d}_1\right)\vec{n}. \tag{16}$$

One can denote $\vec{n}\Delta\vec{d} = m_{ij}$ and take the following expression as interaction micropotential of dust particles

$$\varphi(r) = \frac{eZ}{r}\left(eZ + \frac{m_{ij}}{r}\right). \tag{17}$$

Its Fourier transform has the following form:

$$\tilde{\varphi}(q) = \frac{4\pi e^2 Z}{q^2} - \frac{2\pi^2 eZm_{ij}}{q}.$$

The dielectric function is taken as

$$\varepsilon = 1 + \frac{n_e}{k_BT}\tilde{\phi}_{ee}(q) + \frac{n_d}{k_BT}\tilde{\phi}_{de}(q). \tag{18}$$

Here, $\tilde{\phi}_{ee}(q)$ and $\tilde{\phi}_{ed}(q)$ are the Fourier transforms of interaction micropotentials between electron-electron and electron-dust particle. For potential $\phi_{de}(r) = \frac{e^2Z}{r} - \frac{e\vec{d}\vec{n}}{r^2}$, the Fourier transform for electron—dust particle interaction is

$$\tilde{\phi}_{de}(q) = \frac{4\pi e^2 Z}{q^2} - \frac{2\pi^2 ep_i}{q},$$

where $p_i = \vec{d}\vec{n}$. The Fourier transform for electron—electron interaction is

$$\tilde{\phi}_{ee}(q) = \frac{4\pi e^2}{q^2}.$$

With $\mu = \frac{k_BT}{2\pi^2 n_d ep_i}$, the Fourier transform of the effective screened potential is obtained as

$$\tilde{\Phi}(q) = \frac{\tilde{\varphi}}{\varepsilon} = \frac{4\pi e^2Z^2 + 2\pi^2 eZm_{ij}q}{q^2 - q/\mu + n_dZ/(n_er_D^2) + 1/r_D^2}.$$

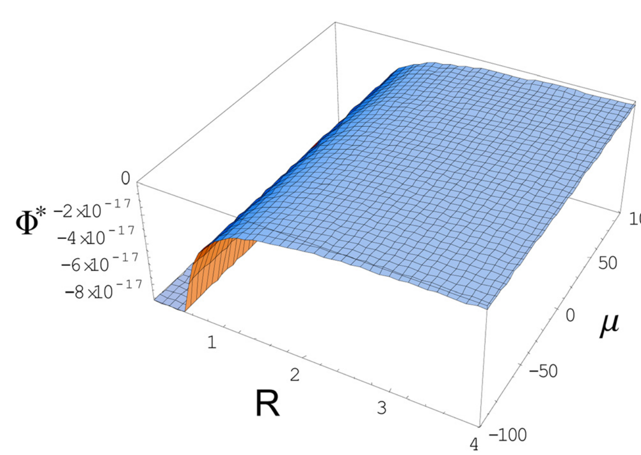

FIG. 3. (Color online) The curve of $\Phi^* = \Phi/k_BT$ obtained by expression (19) as dependence on the distance $R = r/r_{De}$ and parameter μ at $m_{ij} < 0$.

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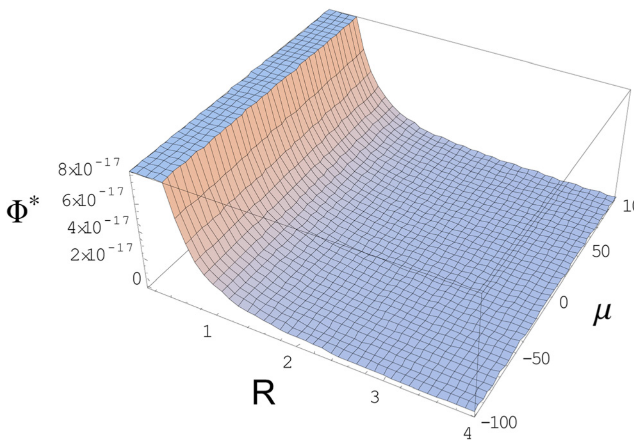

FIG. 4. (Color online) The curve of $\Phi^* = \Phi/k_BT$ obtained by expression (19) as dependence on the distance $R = r/r_{De}$ and parameter μ at $m_{ij} > 0$.

After Fourier transformation, one can obtain the formula for effective interaction potential between two dust particles

$$\Phi(r) = \frac{1}{r}[Ah(K_1 r) + Bh(K_2 r)] + \frac{eZm_{ij}}{r^2}. \qquad (19)$$

For convenience, we define the following function and the coefficients

$$h(ar) = \cos(ar)(\pi + Si(ar)) - Ci(ar)\sin(ar),$$

$$A = 2\pi^2 e^2 Z^2 \left(1 + \frac{1}{\sqrt{1 - 4Z\mu^2/r_D^2}}\right) + \frac{eZm_{ij}}{\mu}\left(1 + \frac{1 - Z\mu^2/r_D^2}{\sqrt{1 - 4Z\mu^2/r_D^2}}\right), \qquad (20)$$

$$B = 2\pi e^2 Z^2 \left(1 + \frac{1}{\sqrt{1 - 4Z\mu^2/r_D^2}}\right) + \frac{eZm_{ij}}{\mu}\left(1 + \frac{1 - Z\mu^2/r_D^2}{\sqrt{1 - 4Z\mu^2/r_D^2}}\right),$$

$$K_{1/2} = \frac{1}{2}\left(1/\mu \pm \sqrt{1/\mu^2 - 4Z/r_D^2}\right).$$

Figs. 3 and 4 show curves $\Phi^* = \Phi/k_BT$ depending on the distance $R = r/r_D$ and μ for the different signs of m_{ij}.

In the case of $m_{ij} < 0$, positive ions in the cloud of first particle and negatively charged second particle are on one side, resulting in predominant attraction, otherwise, when $m_{ij} > 0$, the repulsion prevails. Let us consider case $m_{ij} = 0$. In Fig. 5 is a graph which shows that on the curve of interaction, potential oscillations may occur.

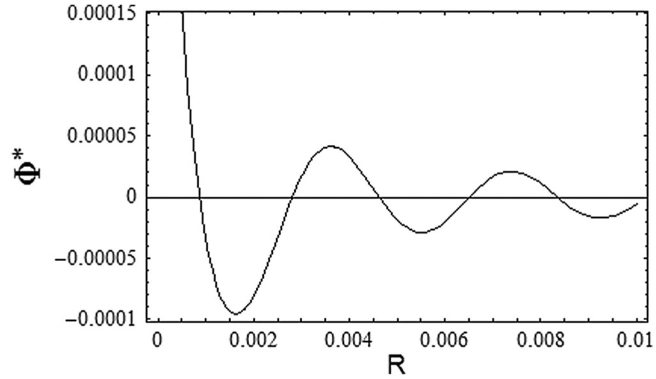

FIG. 5. The curve of $\Phi^* = \Phi/k_BT_e$ obtained by expression (19) as dependence on the distance $R = r/r_{De}$ at $m_{ij} = 0$.

Based on behavior analysis for the effective potentials obtained in present work, one can make the following conclusions:

1. In this paper, the effective interaction potentials of particles in complex plasmas are considered. Pseudopotentials of electron-atom interaction in the classical and semiclassical approximations are obtained. It was shown that with taking into account the effects of diffraction, the effective interaction potential matches to screened classical potential at large distances but differ at small.
2. The interaction of dust particles in plasma-dust structures is considered. The analysis of the character of interaction at various parameters is performed. It is shown that under certain conditions, a force of attraction between dust particles may appear.

For all considered interactions, the method of linear dielectric response is usable.

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