INFLUENCE OF DYNAMIC SCREENING ON THE SCATTERING CROSS SECTIONS OF THE PARTICLES IN THE DENSE NONIDEAL PLASMAS OF NOBLE GASES

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Within the dynamic screening potential model, elastic scattering processes between electrons and atoms in partially ionized plasmas were investigated using the method of phase functions. The phase shifts were calculated by solving the Calogero equation. Differential and total cross sections for the scattering of electrons on noble gas atoms were calculated and compared with experimental and other theoretical data. It was shown that the polarization potential is adequate for description the interaction between charged and neutral particles in partially ionized plasma. Analysis of the results showed that the phase shifts of electron atom scattering obtained with taking into account the dynamic screening are larger than the data obtained with consideration of static screening. The results can be used to calculate the various transport coefficients of the semiclassical dense plasma.

Key words: scattering cross section, dynamic interaction potential, dense nonideal plasma, phase shift, noble gases

1 Introduction

Elastic scattering of electrons on atoms is a fundamental process that continuously attracts the attention of researchers. A large number of various theoretical and experimental studies of elementary processes is devoted to the elastic scattering of electrons on noble gas atoms. Reaction cross section are valuable since they provide information on how particles collide and interact. They are needed to describe transfer processes within partially ionized plasmas. For those practical applications where plasma is a working medium, one needs adequate data on physical properties of the system. Various theoretical methods are used in plasma investigations; one of them employs pseudopotential models that adequately describe the effective interaction between the plasma constituents. In the present work we consider collisional processes in plasmas of complex composition on the basis of the static potential and dynamic potential for the electron-atom interaction. In this work we used the effective potential of electron atom interaction presented in works [1-3] and considering the effects of screening and diffraction:

$$\Phi_{ea}(r) = -\frac{e^2\alpha}{2r^4(1-4\lambda\!\!\!^{-}{}_{ea}{}^2/r_D{}^2)}\left(e^{-Br}(1+Br)-e^{-Ar}(1+Ar)\right)^2, \tag{1}$$

here $A^2 = \left(1+\sqrt{1-4\lambda\!\!\!^{-}{}_{ea}^2/r_D^2}\right)\Big/2\lambda\!\!\!^{-}{}_{ea}{}^2$, $B^2 = \left(1-\sqrt{1-4\lambda\!\!\!^{-}{}_{ea}^2/r_D^2}\right)\Big/2\lambda\!\!\!^{-}{}_{ea}{}^2$. $\lambda\!\!\!^{-}{}_{ea} = \hbar\Big/\sqrt{2\pi\mu_{ea}k_BT} \approx \lambda\!\!\!^{-}{}_e$ is the de Broglie thermal wavelength; $\mu_{ea} = m_em_a/(m_e+m_a)$ is the reduced mass of the atom and the electron; $r_D = \left(k_BT\big/\left(8\pi e^2n_e\right)\right)^{1/2}$ is the Debye length; n_e is the numerical density of electrons; T is the plasma temperature; k_B is the Boltzmanns constant, α is the atomic polarizability. Potential (1) is screened and has finite values at the distances close to zero.

Collision cross sections directly depend on the relative velocity of the colliding particles, contained in the equations for their calculation. The energy of the static interaction usually does not depend on this velocity. This consideration is not entirely correct, and accounting of the influence of the different dynamic effects, in particular, the dynamic screening of the incident charges field, on the interaction energy of the particles is more consistent [4,5]. In work [5] the elastic differential cross sections of charges in a dense semiclassical plasma on the basis of the interaction potential, taking into account the effects of diffraction and the effect of dynamic screening were investigated. In work [6] phase shifts, differential, partial, total and transport scattering cross sections of charges in a nonideal semiclassical plasma on the basis of interaction potential, taking into account the effects of diffraction and the effect of dynamic screening were calculated.

This potential has been obtained by replacing the usual Debye length in the interaction potential (1) by the screening radius, depending on the relative velocity of the colliding particles [7]:

$$r_0 = r_D (1 + \upsilon^2 / \upsilon_{Th}^2)^{1/2} , \tag{2}$$

here υ is the relative velocity of the colliding particles, υ_{Th} is the thermal velocity. Then the energy of the electron-atom interaction, which takes into account dynamic screening, in a dimensionless form is:

$$\Phi_{ea}^{dyn}(r) = -\frac{e^2 \alpha}{2r^4 (1 - 4\lambda_{ea}^2 / r_0^2)} \left(e^{-Br}(1 + Br) - e^{-Ar}(1 + Ar) \right)^2 , \tag{3}$$

where
$A^2 = \left(1 + \sqrt{1 - 4\lambda_{ea}^2 / r_0^2}\right) \Big/ 2\lambda_{ea}^2$, $\quad B^2 = \left(1 - \sqrt{1 - 4\lambda_{ea}^2 / r_0^2}\right) \Big/ 2\lambda_{ea}^2$.

In the case, when the influence of the scattering center is not large, the calculation of the scattering cross sections can be carried out in the Born approximation [8]. In work [5] basing on the results obtained by the Born method, it was shown that dynamic screening increases the differential scattering cross sections in comparison with the Debyes static screening, especially at small scattering angles. At large scattering angles differential cross sections converge.

The method of partial waves [8-9] gives more accurate quantum mechanical solution of the problem of the scattering cross section estimation. In this method the scattering cross sections are calculated on the basis of the scattering phases of the waves with different values of the orbital quantum number. For large values the scattering phase can be found from the semiclassical representation of a particle moving in the field of a fixed force center. Another method for the scattering phase calculation is called as the phase-function method, it reduces to solving of the first order differential equation for the phase function [8].

2 Method and parameters

The method of phase functions is used for the investigation of scattering processes, see Ref. [2,6]. The mathematical background of this method is given by the following fact which is well known in the theory of differential equations: a linear homogeneous equation of the second order (the Schrdinger equation, in our case) can be reduced to a non-linear equation of the first order, i.e. to the Ricatti equation, in our case. The monograph [10] provides a detailed description of this method. Thus, within this approach we solved a first-order differential equation for the scattering phase, i.e. the Calogero equation:

$$\frac{d}{dr}\delta_l(k,r) = -\frac{1}{k}\frac{2m}{\hbar^2}\Phi^{\alpha\beta}(r)\left[\cos\delta_l(k,r)j_l(kr) - \sin\delta_l(k,r)n_l(kr)\right]^2, \quad \delta_l(k,0) = 0, \quad (4)$$

where l is orbital quantum number; $\delta_l(k,0)$ is the phase function; $\Phi^{\alpha\beta}(r)$ is the interaction potential; k is the wave number of the particle; $j_l(kr)$ and $n_l(kr)$ are regular and irregular solutions of the Schrödinger equation. The phase shift is the asymptotical value of the phase function at the large distances:

$$\delta_l(k) = \lim_{r\to\infty}\delta_l(k,r). \quad (5)$$

Note that in the Calogero equation there is an obvious relation between the interaction potential and the scattering phase shift. Besides, it is quite easy to solve this equation numerically. In the present work we employed the Runge-Kutta method of fourth order.

If the scattering phase shifts are known, one can calculate the total cross section for the elastic scattering of plasma particles according to [8]:

$$Q^{\alpha\beta}(k) = \frac{4\pi}{k^2}\sum_{l=0}(2l+1)\sin^2\delta_l^{\alpha\beta}, \quad (6)$$

The wave number is rebated to the energy of the incident particle, $k^2 = \mu_{ea}E/\hbar^2$.

In this work for hydrogen plasma the following dimensionless parameters were used:

$$\Gamma = \frac{e^2}{a\,k_B T}, \quad (7)$$

is the coupling parameter; the average distance between particles is $a = (3/4\pi n)^{1/3}$, $n = n_e + n_i$ density of charged particles;

$$r_s = \frac{a}{a_B}, \quad (8)$$

is the density parameter ($a_B = \hbar^2/m_e e^2$ is the Bohr radius).

3 Results

The results of studies of the collisional processes within the static model (1) have been previously obtained and presented in [11-12]. The comparison with results obtained on the basis of the Debye and Deutsch models, which take into account one of two effects, screening or diffraction, respectively, was presented there. In this paper, we present the data obtained in the framework of the potential (1) for comparison with the results obtained on the basis of a new dynamic interaction potential (3), taking into account the dynamic screening and the effect of diffraction. The goal of this work is to identify the differences in the characteristics of the collisional processes associated with the use of the dynamic screening in the charged particles interaction.

Equation (4) was solved numerically. Figures 1 and 2 show the dependences of the phase function of the electron atom scattering on the distance obtained in the framework of the models of static and dynamic screening. As can be seen in Figures 1 and 2, the obtained phase functions demonstrate proper asymptotic behavior, at large distances they tend to some steady-state value, which is actually the phase shift. Phase shifts calculated within the dynamic potential (3) are larger then phase shifts derived within the static model (1), since the dynamic screening of the field is weaker than the static one.

Figure 3 shows the dependences of the differential cross sections of the electron-atom scattering on the scattering angle obtained in the framework of the models of static and dynamic screening. As can be seen in Figure 3, the differential cross sections calculated within the dynamic potential (3) lay higher then differential cross sections derived from the static model (1).

Figures 4 and 5 present the total cross sections for the scattering on the basis of static and dynamic potentials of the electrons on neutral He, Ne, respectively. The comparison of our results with the data obtained in experimental and other theoretical works shows that the dynamic potential used in the present work describes adequately the interaction of the electrons with atoms.

4 Conclusion

Based on the dynamic model of the charged particles interaction in nonideal semiclassical plasma phase shifts, differential and total scattering cross sections of the particles of noble gases were investigated. Quantum mechanical method has been used for their calculation. Analysis of the results showed that the phase shifts of electron atom scattering obtained with taking into account the dynamic screening are larger than the data obtained with consideration of static screening. The comparison of our results with the data obtained in the experiments and other theoretical works shows that the dynamic potential used in the present work adequately described the elastic scattering of the electron on atoms of noble gases.

The results can be used to calculate the various transport coefficients of the semiclassical dense plasma. Knowledge of these characteristics plays a major role in the design of technical installations associated with the use of the dense nonideal plasma, for example, inertial confinement fusion facilities.

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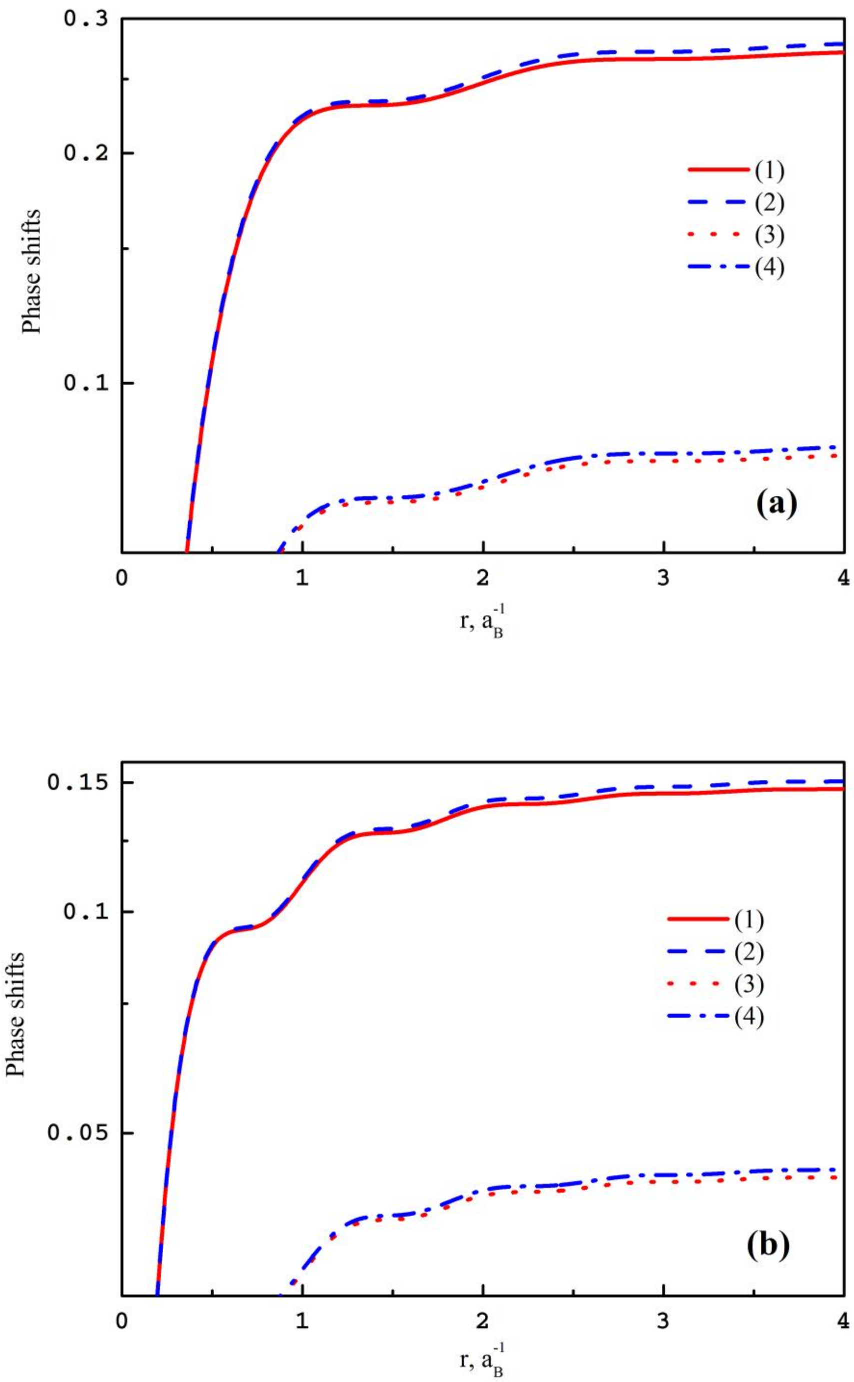
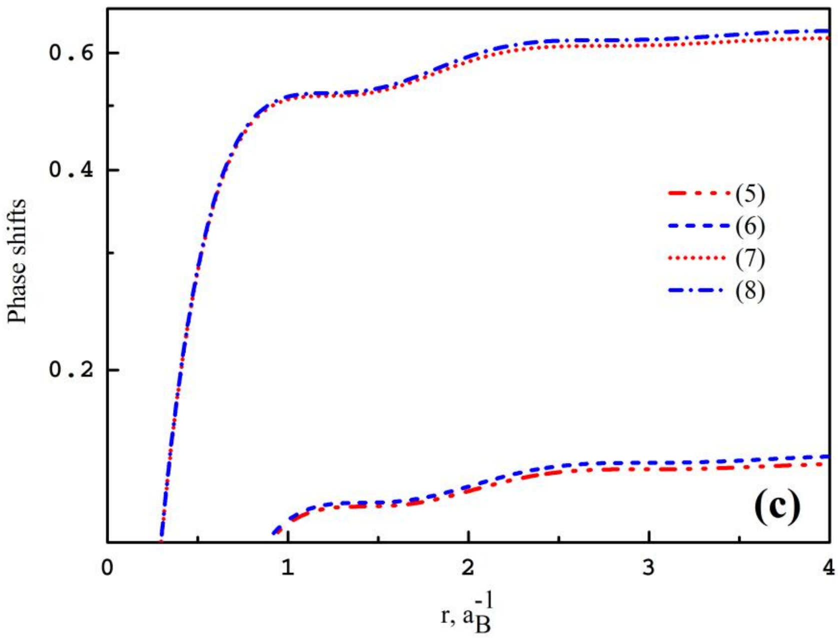


Figure 1 – Phase functions of electron-atom scattering in the dense plasma of noble gases. a) $\Gamma = 0.5$, $r_s = 10$, $k = 2a_B^{-1}$. 1 - On the basis of the static potential on the hydrogen atom; 2 - On the basis of the dynamic potential on the hydrogen atom; 3 - On the basis of the static potential on the helium atom; 4 - On the basis of the dynamic potential on the helium atom; b) $\Gamma = 0.5$, $r_s = 10$, $k = 4a_B^{-1}$. 1 - On the basis of the static potential on the hydrogen atom; 2 - On the basis of the dynamic potential on the hydrogen atom; 3 - On the basis of the static potential on the helium atom; 4 - On the basis of the dynamic potential on the helium atom;



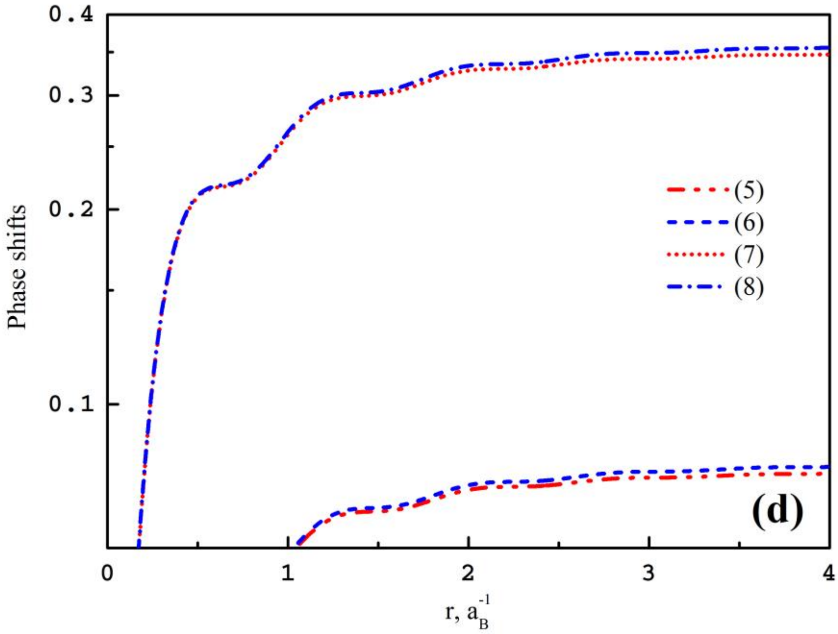


Figure 2 – Phase functions of electron-atom scattering in the dense plasma of noble gases. c) $\Gamma = 0.5$, $r_s = 10$, $k = 2a_B^{-1}$. 5 - On the basis of the static potential on the neon atom; 6 - On the basis of the dynamic potential on the neon atom; 7 – On the basis of the static potential on the argon atom; 8 - On the basis of the dynamic potential on the argon atom; d) $\Gamma = 0.5$, $r_s = 10$, $k = 4a_B^{-1}$. 5 - On the basis of the static potential on the neon atom; 6 - On the basis of the dynamic potential on the neon atom; 7 - On the basis of the static potential on the argon atom; 8 - On the basis of the dynamic potential on the argon atom;

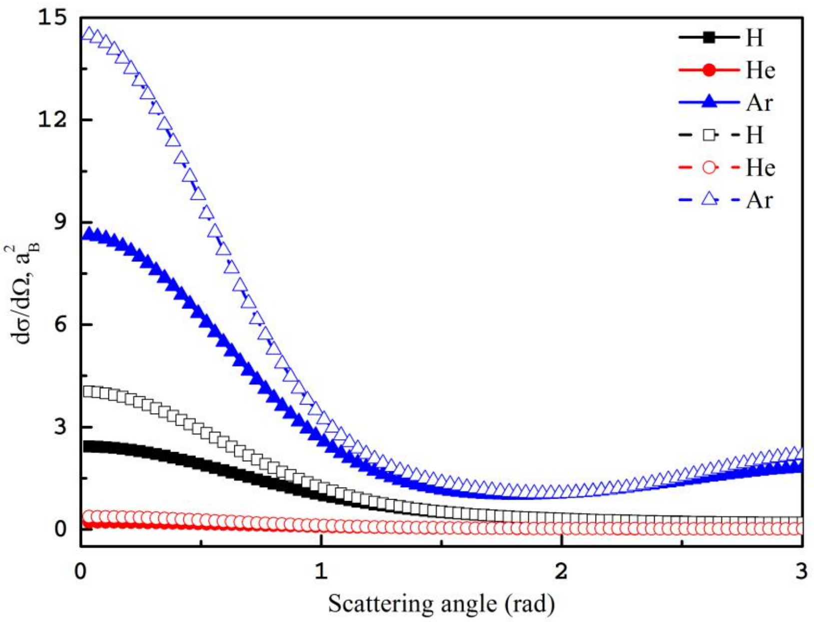


Figure 3 – Differential cross sections of the electron-atom scattering depending on the scattering angle, $\Gamma = 0.7$, $r_s = 4$, $k = 0.8a_B^{-1}$. Open symbol - on the basis of the dynamic potential; Solid symbol - on the basis of the static potential;

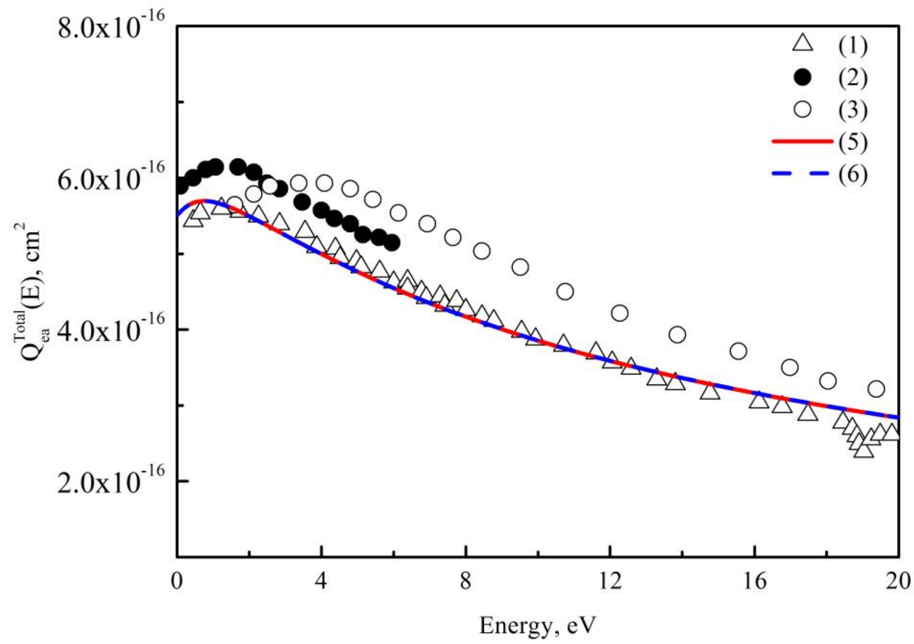


Figure 4 – Total cross sections for electron scattering on the helium atom on the basis of static and dynamic potentials. 1 - [13]; 2 - [14]; 3 - [15]; 4 - on the basis of the dynamic potential; 5 - on the basis of the static potential.

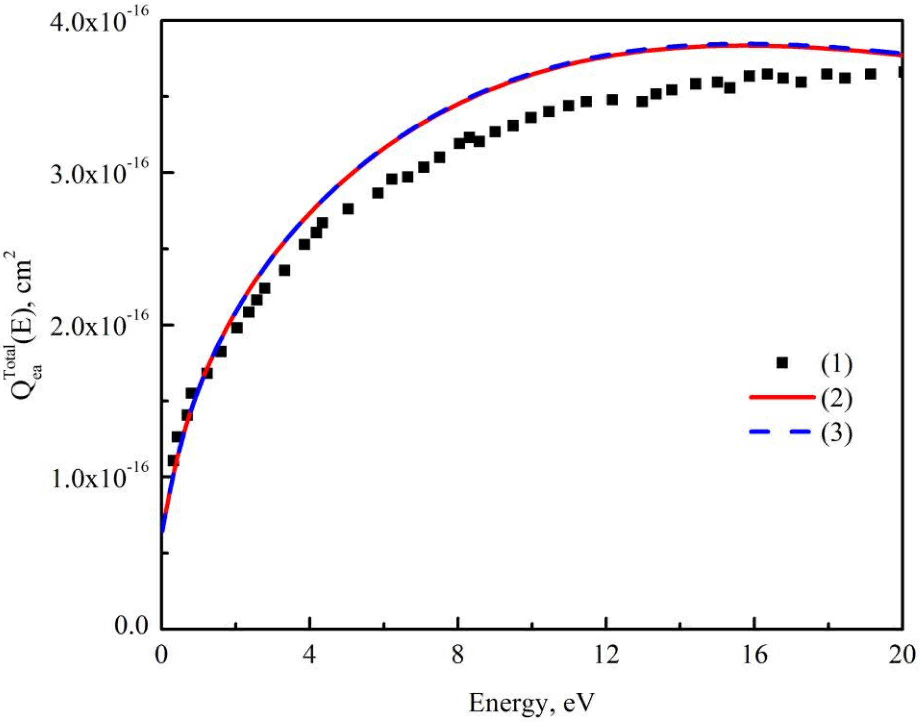


Figure 5 – Figure 4 - Total cross sections for electron scattering on the neon atom on the basis of static and dynamic potentials. 1 - [16]; 2 - on the basis of the static potential; 3 - on the basis of the dynamic potential.