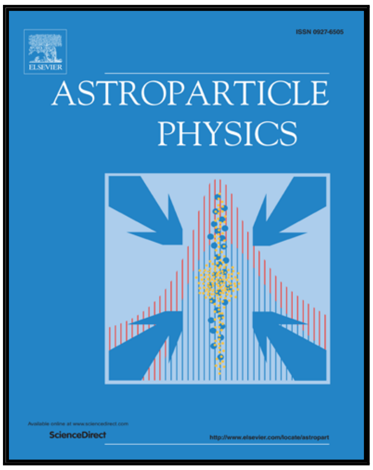
Accepted Manuscript



Some astrophysical effects of nonlinear vacuum electrodynamics in magnetosphere of pulsar

Medeu Abishev, Yerlan Aimuratov, Yermek Aldabergenov, Nurzada Beissen, Zhami Bakytzhan, Meruert Takibayeva

PII: S0927-6505(15)00080-8
DOI: 10.1016/j.astropartphys.2015.04.010
Reference: ASTPHY 2047

To appear in: *Astroparticle Physics*

Received date: 4 February 2015
Revised date: 6 April 2015
Accepted date: 29 April 2015

Please cite this article as: Medeu Abishev, Yerlan Aimuratov, Yermek Aldabergenov, Nurzada Beissen, Zhami Bakytzhan, Meruert Takibayeva, Some astrophysical effects of nonlinear vacuum electrodynamics in magnetosphere of pulsar, *Astroparticle Physics* (2015), doi: 10.1016/j.astropartphys.2015.04.010

Some astrophysical effects of nonlinear vacuum electrodynamics in magnetosphere of pulsar

Medeu Abishev[a,], Yerlan Aimuratov[a,b], Yermek Aldabergenov[a], Nurzada Beissen[a], Zhami Bakytzhan[a], and Meruert Takibayeva[a]

[a]*al-Farabi Kazakh National University, Al-Farabi av. 71, 050038, Almaty, Kazakhstan*
[b]*Fessenkov Astrophysical Institute, Observatory 23, 050020, Almaty, Kazakhstan*

Abstract

It has been considered the propagation of hard electromagnetic emissions in magnetosphere of pulsar on the base of General Relativity and nonlinear vacuum electrodynamics. It is shown that the radiation will propagate having different velocities in magnetosphere of pulsar and has form of two normal modes polarized in mutually orthogonal planes. It is calculated the delay between the two orthogonal modes, as they propagate from the pulsar to the detecting device.

Keywords: gamma ray astrophysics, pulsar, quantum electrodynamics, General Relativity

1. Introduction

The theory [1] and the experiment [2] shows that electrodynamics in vacuum is nonlinear. Therefore, researches [3-10] on various manifestations of nonlinearity in electrodynamics are of undoubted interest.

In a number of recent studies [11-19] various effects of nonlinear vacuum electrodynamics has been considered along with the possibility of their measurement in the laboratory. However, due to the fact that magnetic fields that can be created in the laboratory $B \sim 10^5$ G, are significantly smaller than the quantum value $B_q = m^2c^3/(e\hbar) = 4.41 \times 10^{13}$ G, their observation

Email address: abishevme@gmail.com (Medeu Abishev)

will only be possible in the future, after further development of measurement technology. Therefore, at present the main interest lies in the astrophysical effects of nonlinear vacuum electrodynamics occurring in the magnetic fields $B \sim B_q$ of pulsars and collapsars

For the first time the effect of nonlinear corrections of the vacuum electrodynamics to the polarization and directivity of the radiation of X-ray pulsars have been studied in [20-22]. The calculations show that the electromagnetic wave passing through the magnetic field of the pulsar should feel nonlinear electrodynamic birefringence; it should be split into two normal modes having mutually orthogonal polarization and distributing via non-coincident rays at different speeds. But in the magnetosphere of pulsar X-rays should feel and birefringence also due to the presence of plasma in it.

As shown in [22], at a certain concentration of plasma the birefringence caused by nonlinear vacuum electrodynamics exceeds the one induced by plasma. For gamma radiation, due to its higher frequency, the distorting effect of the plasma is not noticeable. Therefore, the information on distribution and density of the field in the emitting region can be obtained by studying the polarization states of the hard radiation of pulsars [22].

Later, these studies were continued in [23-30]. For some special cases (propagation of electromagnetic wave in the plane of the magnetic equator and the magnetic meridian of pulsars) in [27-29] authors show that the main nonlinear electrodynamic effect, which can be registered on Earth, is difference between the velocities of the normal modes in magnetic field of the pulsar [29-30]. However, in previously published studies [24-25, 27], such calculation was carried out in only a few simple cases.

So let's make a calculation of this effect in the most general case, when an electromagnetic pulse passes through the magnetic field of the pulsar in a random direction. Suppose that at some point in the magnetosphere or on surface of pulsar there was a relatively short burst of hard radiation. We also assume that electromagnetic pulse emerged during this outbreak is either unpolarized or has an elliptical polarization [21].

In a strong magnetic field of pulsar the radiation pulse, due to the birefringence, splits into two pulses polarized in mutually perpendicular planes and having different velocities. Therefore, the two electromagnetic pulses emitted at the same time from the same source will arrive to the recording device installed on the near-Earth satellite over different beams at different times $t_2 \neq t_1$. This device will first register an arrival of the front part of a more rapid pulse and the polarization of the detected radiation will be linear over time $t_2 - t_1$. After this period of time the front part of the second pulse having an orthogonal polarization will come to the recorder. Therefore, the further polarization of the total momentum will be arbitrary. For observation of this effect detectors of electromagnetic radiation of pulsars must be equipped with devices that would measure the polarization state of the radiation.

The study of the manifestation of nonlinear electrodynamic birefringence in the general case, when a relatively short electromagnetic pulse passes the magnetic field of the pulsar in an arbitrary direction, as well as the calculation of period $t_2 - t_1$ are the new information obtained in the present study.

2. The equations of nonlinear vacuum electrodynamics and gravitation

Consider nonlinear post-Maxwell electrodynamics, which is a direct consequence of quantum electrodynamics [1]. Its Lagrangian [31] has the form:

$$L = \frac{\sqrt{-g}}{32\pi}\Big\{2I_2 + \xi\Big[(\eta_1 - 2\eta_2)I_2^2 + 4\eta_2 I_4\Big]\Big\} - \frac{\sqrt{-g}}{c}j^\beta A_\beta,$$

where j^β is the current density four-vector, g - determinant of the metric tensor, $\xi = 1/B_q^2$, $I_2 = F_{\beta\sigma}F^{\sigma\beta}$ and $I_4 = F_{\beta\sigma}F^{\sigma\nu}F_{\nu\mu}F^{\mu\beta}$ – invariants of the electromagnetic tensor $F_{\beta\sigma}$ and according to quantum electrodynamics $\eta_1 = e^2/(45\pi\hbar c) = 5.1 \times 10^{-5}$, $\eta_2 = 7e^2/(180\pi\hbar c) = 9.0 \times 10^{-5}$.

The field equations derived from this Lagrangian have the form:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\beta}\Big\{\sqrt{-g}Q^{\sigma\beta}\Big\} = -\frac{4\pi}{c}j^\sigma, \tag{2.1}$$

$$Q^{\sigma\beta} = 8\pi\frac{\partial L}{\partial F_{\beta\sigma}} = \Big\{1 + \xi(\eta_1 - 2\eta_2)I_2\Big\}F^{\sigma\beta} + 4\xi\eta_2 F^{\sigma\nu}F_{\nu\mu}F^{\mu\beta}.$$

The second pair of equations of electrodynamics coincides with the corresponding equations of Maxwell's theory:

$$\frac{\partial F_{\mu\beta}}{\partial x^\nu} + \frac{\partial F_{\beta\nu}}{\partial x^\mu} + \frac{\partial F_{\nu\mu}}{\partial x^\beta} = 0. \tag{2.2}$$

Metric tensor in equations (2.2) satisfy Einstein equations [32]:

$$R_{\beta\sigma} - \frac{1}{2}g_{\beta\sigma}R = -\frac{8\pi G}{c^4}T_{\beta\sigma}, \tag{2.3}$$

where $R_{\beta\sigma} = R^{\nu}_{\beta\sigma\nu}$ – Ricci tensor, $T_{\beta\sigma}$ – energy-momentum tensor of the matter and all fields, including electromagnetic. The system of equations (2.1) - (2.3) in our problem will be sought by the method of successive approximations with a precision linear in the small dimensionless parameters: the gravitational potential and post-Maxwell amendments. The gravitational field of the pulsar will be assumed to be spherically symmetric, and in the harmonic Fock coordinates [32] metric will be expanded in the small parameter α/r with the required accuracy:

$$g_{00} = 1 - \frac{2\alpha}{r}, \quad g_{rr} = -1 - \frac{2\alpha}{r}, \quad g_{\theta\theta} = r^2 g_{rr}, \quad g_{\phi\phi} = g_{\theta\theta}\sin^2\theta, \tag{2.4}$$

where $\alpha = \gamma M/c^2$, γ – gravitational constant, and M – mass of the pulsar.

Suppose that at time $t = 0$ from the point $\mathbf{r} = \mathbf{r}_0$ of the pulsar magnetosphere hard radiation impulse was emitted. Then, in magnetic field of the pulsar that impulse, because of birefringence, will split [33] into two impulses with orthogonal polarizations and moving at different speeds.

For the convenience of further calculations, we introduce the spherical coordinate system as follows. Consider a beam of the first normal mode and draw a tangent to it at the point $\mathbf{r} = \mathbf{r}_0$. Axis of the spherical coordinate system will be directed in such a way, so that the tangent to the chosen beam and the center of the pulsar would be lying in the same plane, and $\theta = \pi/2$, and the azimuthal coordinate ϕ of the source of hard radiation would be equal to $\phi = 0$.

Without loss of generality, we assume that in this coordinate system, the vector of the magnetic dipole moment of the pulsar $\mathbf{m}$ is directed to a point with spherical coordinates θ_0 and ϕ_0. Then the Cartesian components of the magnetic dipole moment $\mathbf{m}$ take the form:

$$m_x = |\mathbf{m}|\sin\theta_0\cos\phi_0, \quad m_y = |\mathbf{m}|\sin\theta_0\sin\phi_0, \quad m_z = |\mathbf{m}|\cos\theta_0.$$

As it is accepted [34] in the problems of celestial mechanics, instead of the radial coordinate r we introduce the coordinate $u = 1/r$. Then, the non-zero components of the dipole electromagnetic field tensor of the pulsar, in the coordinate system u, θ, ϕ, with the required for our purposes accuracy will be:

$$F_{u\theta} = -F_{\theta u} = |\mathbf{m}|\sin\theta_0\sin(\phi - \phi_0),$$
$$F_{u\phi} = -F_{\phi u} = |\mathbf{m}|\sin\theta[\sin\theta_0\cos\theta\cos(\phi - \phi_0) - \sin\theta\cos\theta_0], \tag{2.5}$$

$$F_{\phi\theta} = -F_{\theta\phi} = 2|\mathbf{m}|u\sin\theta[\sin\theta_0\sin\theta\cos(\phi-\phi_0)+\cos\theta\cos\theta_0].$$

In electrodynamics when solving most of the problems the eikonal method [32-33, 35-36] is used, which allows studying the motion of electromagnetic impulses by their beams. Application of this method to nonlinear electrodynamics has shown [33,37] that the propagation of electromagnetic waves in external electromagnetic and gravitational fields in nonlinear electrodynamics with field equations (2.1)-(2.3) is equivalent to the propagation of the normal modes through the isotropic geodesics in effective space-time for which metric tensor $G^{eff(1,2)}_{\nu\mu}$ has the form:

$$G^{eff\ (1,2)}_{\nu\mu} = g_{\nu\mu} - 4\eta_{(1,2)}\xi F_{\nu\beta}g^{\beta\sigma}F_{\sigma\mu}. \tag{2.6}$$

Therefore the study of the laws of propagation of electromagnetic impulses in magnetic (2.5) and gravitational (2.4) fields of a pulsar is conveniently carried out not by using equations (2.1)-(2.2), but based on the analysis of isotropic geodesics in space-time with the metric tensor (6).

Let us substitute expressions (2.4) and (2.5) into (2.6) and write down the components of the metric tensor of the effective space-time $G^{eff\ (1,2)}_{\nu\mu} \equiv G^{(1,2)}_{\nu\mu}$ explicitly:

$$G^{(1,2)}_{00} = 1 - 2\alpha u,$$

$$G^{(1,2)}_{uu} = -\frac{1+2\alpha u}{u^4} - 4\mathbf{m}^2\xi\eta_{1,2}u^2\{\sin^2\theta_0\sin^2(\phi-\phi_0)+$$
$$+[\sin\theta_0\cos\theta\cos(\phi-\phi_0)-\sin\theta\cos\theta_0]^2\},$$

$$G^{(1,2)}_{u\theta} = 8\mathbf{m}^2\xi\eta_{1,2}u^3[\sin\theta_0\sin\theta\cos(\phi-\phi_0)+\cos\theta\cos\theta_0]\times$$
$$\times[\sin\theta_0\cos\theta\cos(\phi-\phi_0)-\sin\theta\cos\theta_0],$$

$$G^{(1,2)}_{u\phi} = -8\mathbf{m}^2\xi\eta_{1,2}u^3[\sin\theta_0\sin\theta\cos(\phi-\phi_0)+\cos\theta\cos\theta_0]\sin\theta\sin\theta_0\sin(\phi-\phi_0),$$

$$G^{(1,2)}_{\theta\theta} = -\frac{(1+2\alpha u)}{u^2} - 4\mathbf{m}^2\xi\eta_{1,2}u^4\{\sin^2\theta_0\sin^2(\phi-\phi_0)+$$
$$+4[\sin\theta_0\sin\theta\cos(\phi-\phi_0)+\cos\theta\cos\theta_0]^2\},$$

$$G^{(1,2)}_{\theta\phi} = -4\mathbf{m}^2\xi\eta_{1,2}u^4[\sin\theta_0\cos\theta\cos(\phi-\phi_0)-\sin\theta\cos\theta_0]\sin\theta\sin\theta_0\sin(\phi-\phi_0),$$

$$G^{(1,2)}_{\phi\phi} = -\Big\{\frac{(1+2\alpha u)}{u^2} + 4\mathbf{m}^2\xi\eta_{1,2}u^4\{[\sin\theta_0\cos\theta\cos(\phi-\phi_0)-$$
$$-\sin\theta\cos\theta_0]^2 + 4[\sin\theta_0\sin\theta\cos(\phi-\phi_0)+\cos\theta\cos\theta_0]^2\}\Big\}\sin^2\theta. \tag{2.7}$$

Equations for isotropic geodesics in the effective space-time with the metric tensor (2.7) will be written in the form where differentiation is performed not with respect to the affine parameter σ, but with respect to the azimuthal angle ϕ :

$$\begin{aligned}
&\frac{d^2x^0}{d\phi^2} + \{\Gamma^0_{\beta\mu} - \frac{dx^0}{d\phi}\Gamma^3_{\beta\mu}\}\frac{dx^\beta}{d\phi}\frac{dx^\mu}{d\phi} = 0, \\
&\frac{d^2u}{d\phi^2} + \{\Gamma^1_{\beta\mu} - \frac{du}{d\phi}\Gamma^3_{\beta\mu}\}\frac{dx^\beta}{d\phi}\frac{dx^\mu}{d\phi} = 0, \\
&\frac{d^2\theta}{d\phi^2} + \{\Gamma^2_{\beta\mu} - \frac{d\theta}{d\phi}\Gamma^3_{\beta\mu}\}\frac{dx^\beta}{d\phi}\frac{dx^\mu}{d\phi} = 0,
\end{aligned} \tag{2.8}$$

where $\Gamma^\nu_{\beta\mu}$ – Christoffel symbols defined in effective space-time with the metric tensor (2.7).

The system of equations (2.8) has a first integral:

$$G^{(1,2)}_{\beta\mu}\frac{dx^\beta}{d\phi}\frac{dx^\mu}{d\phi} = 0. \tag{2.9}$$

Equations (2.8) and (2.9) are non-linear, but in our case there are small parameters αu and $\mathbf{m}^2\xi\eta_{1,2}u^6$. Therefore, the solution of these equations will be sought by the method of successive approximations in these small parameters. Since the angles of nonlinear-electrodynamic bending of the beams of the first and second normal modes are different even in the meridian and equatorial planes [27-29], the boundary conditions for them should be different to ensure that the two impulses arrive in the same detector located on the near-Earth spacecraft [38] Let us require the chosen beam of the first normal mode to start at $u = u_0 = 1/r_0,\ \theta = \pi/2,\ \phi = 0$ at $t = 0$, and the coordinate u at the pericenter to be equal to some given value $u_p = 1/r_p > u_0$, where r_0 – distance from the center of the pulsar to the source of high-energy radiation, r_p – distance from the pulsar to the pericenter of the beam. We also assume that $r_p > R_n$, where R_n – radius of the neutron star.

Another condition follows from the formulation of the problem: if at chosen orientation of the coordinate axes the beam starts from the point $\phi = 0,\ u = u_0,\ \theta = \pi/2$ touching the plane $\theta = \pi/2$. It follows that for the beam of the first normal mode at $\phi = 0$ the following must be satisfied: $d\theta/d\phi = 0$.

3. Solution of the equations for beams of the first normal mode

In the zeroth approximation in small parameters the beam under mentioned boundary conditions, will be a straight line in the plane $\theta = \pi/2$, passing through the point $u = u_0$, $\phi = 0$ and take a value $u = u_p$ at pericenter. It follows that in this approximation the expression $d\theta/d\phi = 0$ for all point of the selected beam and the system of equations (2.8)-(2.9) takes the next form:

$$\frac{d^2x^0}{d\phi^2} = -\frac{2}{u}\left(\frac{du}{d\phi}\right)\left(\frac{dx^0}{d\phi}\right), \quad \frac{d^2u}{d\phi^2} = -u, \quad u^4\left(\frac{dx^0}{d\phi}\right)^2 - u^2 - \left(\frac{du}{d\phi}\right)^2 = 0.$$

Solving this system of equations with the boundary conditions, we arrive at the relations:

$$u(\phi) = u_p \sin(\phi + \psi),$$
$$x^0(\phi) = ct = \frac{\cos\psi}{u_p \sin\psi} - \frac{\cos(\phi+\psi)}{u(\phi)}, \tag{3.1}$$
$$\theta(\phi) = \frac{\pi}{2},$$

where ψ is defined from: $\sin\psi = u_0/u_p$. Since within the magnetosphere $r_0 < 100R_n$, then the angle ψ meets the conditions: $0 < \psi < \pi$.

Using an expression (3.1), we search the solution of the system of equations (2.8)-(2.9) for the first normal wave in the ordinary form for equations of that type:

$$u(\phi) = u_p \sin(\phi+\psi) + \alpha u_p^2 \Phi_1(\phi) + \mathbf{m}^2 \xi \eta_1 u_p^7 \Phi_2(\phi),$$
$$\theta(\phi) = \frac{\pi}{2} + \alpha u_p \Phi_3(\phi) + \mathbf{m}^2 \xi \eta_1 u_p^6 \Phi_4(\phi), \tag{3.2}$$
$$x^0(\phi) = \frac{\cos\psi}{u_p \sin\psi} - \frac{\cos(\phi+\psi)}{u(\phi)} + \alpha \Phi_5(\phi) + \mathbf{m}^2 \xi \eta_1 u_p^5 \Phi_6(\phi),$$

where $\Phi_a(\phi)$, $a = 1-6$ are unknown functions of the azimuthal angle ϕ, having zero order of smallness.

Substituting (3.2) into the left-hand sides of equations (2.8), expanding them in the small parameters to first order inclusive, we obtain the following equation for u :

$$\mathbf{m}^2 \xi \eta_1 u_p^7 \Big\{ \Phi_2'' + \Phi_2 - 6\sin^4(\phi+\psi)\{2\sin(\phi+\psi) + \Big[14\sin(\phi+\psi) -$$

$$-\sin 2(\phi-\phi_0)\cos(\phi+\psi)[7\sin^2(\phi+\psi)-3]+2\sin^2(\phi-\phi_0)\sin(\phi+\psi)\times$$

$$\times[8\sin^2(\phi+\psi)-9]-12\sin^3(\phi+\psi)\Big]\sin^2\theta_0\}\Big\}+\alpha u_p^2\Big\{\Phi_1''+\Phi_1-2\Big\}=0. \quad (3.3)$$

The equation for determining the angle θ takes the form:

$$\mathbf{m}^2\xi\eta_1 u_p^6\Big\{\Phi_4''+\Phi_4+6\sin^4(\phi+\psi)[3\cos(\phi-\phi_0)\cos^2(\phi+\psi)-$$

$$-\sin(\phi-\phi_0)\sin 2(\phi+\psi)]\sin 2\theta_0\Big\}+\alpha u_p\Big\{\Phi_3''+\Phi_3\Big\}=0. \quad (3.4)$$

We now write the first integral (2.9) in this approximation:

$$\mathbf{m}^2\xi\eta_1 u_p^5\Big\{-\Phi_6'-2\sin^4(\phi+\psi)\Big[\sin 2(\phi+\psi)\sin 2(\phi-\phi_0)+$$

$$+4\sin^2(\phi+\psi)+[1-5\sin^2(\phi+\psi)]\sin^2(\phi-\phi_0)\Big]\sin^2\theta_0-$$

$$-2\sin^4(\phi+\psi)\cos^2\theta_0\Big\}-2\alpha\Big\{\Phi_5'\sin(\phi+\psi)+2\Big\}=0. \quad (3.5)$$

Equation for determining the x^0, we will not write, as it is a consequence of (3.3)-(3.5).

Fundamental system of solutions of homogeneous equations for the system (3.3)-(3.4) can be conveniently represented in the form: $y_1=\sin(\phi+\psi)$, $y_2=\cos(\phi+\psi)$. Wronskian [39] of this system: $W=y_1'y_2-y_2'y_1=1$. Therefore, the general solution of these equations has the form:

$$\Phi_1(\phi)=2+S_1\sin\phi+C_1\cos\phi,$$

$$\Phi_2(\phi)=\frac{1}{64}\Big\{f_2(\phi)+S_2\sin\phi+C_2\cos\phi\Big\},$$

$$\Phi_3(\phi)=S_3\sin\phi+C_3\cos\phi,$$

$$\Phi_4(\phi)=\frac{\sin 2\theta_0}{64}\Big\{f_4(\phi)+S_4\sin\phi+C_4\cos\phi\Big\}, \quad (3.6)$$

where $S_1,\ S_2,\ S_3,\ S_4,\ C_1,\ C_2,\ C_3,\ C_4$ are integration constants and for the convenience of further calculations we use the notation:

$$f_2(\phi)=\sin^2\theta_0\Big\{\cos 2(\phi_0+\psi)\Big[195\phi\cos(\phi+\psi)+65\sin^3(\phi+\psi)+26\sin^5(\phi+\psi)+$$

$$+152\sin^7(\phi+\psi)-144\sin^9(\phi+\psi)\Big]+2\sin 2(\phi_0+\psi)\Big[[72\sin^8(\phi+\psi)-40\sin^6(\phi+\psi)-$$

$$-26\sin^4(\phi+\psi)-39\sin^2(\phi+\psi)]\cos(\phi+\psi)+39\phi\sin(\phi+\psi)\Big]+32\sin^7(\phi+\psi)-$$

$$-24\sin^5(\phi+\psi)-60\sin^3(\phi+\psi)-180\phi\cos(\phi+\psi)\Big\}-$$

$$-16\Big[2\sin^5(\phi+\psi)+5\sin^3(\phi+\psi)+15\phi\cos(\phi+\psi)\Big],$$

$$f_4(\phi)=\Big[75\phi\cos(\phi+\psi)+25\sin^3(\phi+\psi)+10\sin^5(\phi+\psi)-40\sin^7(\phi+\psi)\Big]\sin(\phi_0+\psi)+$$

$$+\Big[3\phi\sin(\phi+\psi)-[3\sin^2(\phi+\psi)+2\sin^4(\phi+\psi)+40\sin^6(\phi+\psi)]\cos(\phi+\psi)\Big]\cos(\phi_0+\psi).$$

Solving the equations for functions $\Phi_5(\phi)$ and $\Phi_6(\phi)$, belong to (3.5), we have:

$$\Phi_5(\phi)=A_5-2\ln\left|\frac{[1-\cos(\phi+\psi)]}{\sin(\phi+\psi)}\right|,\quad \Phi_6(\phi)=\frac{1}{64}\Big\{A_6+f_6(\phi)\Big\},\qquad (3.7)$$

where A_5, A_6 are integration constants and as shorthand used:

$$f_6(\phi)=\Big\{16\sin 2(\phi_0+\psi)\sin^6(\phi+\psi)[4-9\sin^2(\phi+\psi)]-\cos 2(\phi_0+\psi)\times$$

$$\times\Big[\{144\sin^7(\phi+\psi)+8\sin^5(\phi+\psi)+26\sin^3(\phi+\psi)+39\sin(\phi+\psi)\}\cos(\phi+\psi)-39\phi\Big]+$$

$$+4[8\sin^5(\phi+\psi)+6\sin^3(\phi+\psi)+9\sin(\phi+\psi)]\cos(\phi+\psi)-36\phi\Big\}\sin^2\theta_0+$$

$$+8\Big[3+2\sin^2(\phi+\psi)\Big]\sin 2(\phi+\psi)-48\phi.$$

By virtue of boundary conditions the functions $\Phi_1(\phi)$ and $\Phi_2(\phi)$ for beams of the first normal mode should become zero at $\phi=0$ and $\phi=\pi/2-\psi$, and constants of the expressions (3.6) will take the following values:

$$C_1=-2,\quad S_1=-\frac{2\cos\psi}{(1+\sin\psi)},\quad C_2=-f_2(0),$$

$$S_2=f_2(0)\mathrm{tg}\psi-\frac{1}{\cos\psi}\Big\{\Big[99\cos 2(\phi_0+\psi)+39(\pi-2\psi)\sin 2(\phi_0+\psi)-52\Big]\sin^2\theta_0-112\Big\}.\qquad (3.8)$$

By the choice of orientation the axes of the spherical coordinate system, the boundary conditions for the functions Φ_3 and Φ_4 have next form:

$$\Phi_3(0)=\Phi_4(0)=0,\quad \frac{d\Phi_3}{d\phi}_{|\phi=0}=\frac{d\Phi_4}{d\phi}_{|\phi=0}=0.\qquad (3.9)$$

Hence $S_3 = C_3 = 0$ and therefore $\Phi_3 = 0$. The constants C_4 and S_4 according to (3.9) should have the form:

$$C_4 = -f_4(0), \quad S_4 = 5\sin(\phi_0+\psi)\cos\psi\Big[56\sin^6\psi - 10\sin^4\psi -$$

$$-15\sin^2\psi - 15\Big] + \cos(\phi_0+\psi)\sin\psi\Big[3 - 280\sin^6\psi + 230\sin^4\psi - \sin^2\psi\Big]. \quad (3.10)$$

The constants A_5 and A_6 are found from the condition $\Phi_5(\phi) = \Phi_6(\phi) = 0$ at $\phi = 0$. Then we obtain form (3.7):

$$A_5 = 2\ln\left|\frac{1-\cos\psi}{\sin\psi}\right|, \quad A_6 = -f_6(0) \qquad (3.11).$$

The result of the expressions (3.2) will take the form:

$$u(\phi) = u_p\sin(\phi+\psi) - 2\alpha u_p^2[\cos\phi + \frac{\sin\phi\cos\psi}{(1+\sin\psi)} - 1] +$$

$$+\frac{\mathbf{m}^2\xi\eta_1 u_p^7}{64}\Big\{f_2(\phi) + \Big[f_2(0)\mathrm{tg}\psi - \frac{1}{\cos\psi}\{[99\cos 2(\phi_0+\psi) +$$

$$+39(\pi - 2\psi)\sin 2(\phi_0+\psi) - 52]\sin^2\theta_0 - 112\}\Big]\sin\phi - f_2(0)\cos\phi\Big\},$$

$$\theta(\phi) = \frac{\pi}{2} + \frac{\mathbf{m}^2\xi\eta_1 u_p^6\sin 2\theta_0}{64}\Big\{f_4(\phi) - f_4(0)\cos\phi + S_4\sin\phi\Big\},$$

$$x^0(\phi) = \frac{\cos\psi}{u_p\sin\psi} - \frac{\cos(\phi+\psi)}{u(\phi)} + 2\alpha\Big\{\ln\left|\frac{1-\cos\psi}{\sin\psi}\right| -$$

$$-\ln\left|\frac{1-\cos(\phi+\psi)}{\sin(\phi+\psi)}\right|\Big\} + \frac{\mathbf{m}^2\xi\eta_1 u_p^5}{64}\Big\{f_6(\phi) - f_6(0)\Big\}. \qquad (3.12)$$

The beam of the first normal mode after exiting the vicinity of the pulsar has to be detected by measuring device located in Earth orbit. Since the nearest pulsars locate [40] at considerable distance ($r \sim 10$ kps $>> R_n$) from the Earth, it is possible to assume that in the chosen coordinate system our measuring device has the coordinate $u_1 = 1/r_1 << u_p$. This condition allows everyone to simply define the required angular coordinates ϕ_1 and θ_1 of the device with an aim to register the beam of the first normal mode. We assume $\phi_1 = \pi - \psi + \beta_1$, where $\beta_1 << 2\pi$.

Substituting this value of ϕ_1 in the equation $u(\phi_1) = u_1$, and deriving it up to the first order with respect to β_1, we will have:

$$\beta_1 = -\frac{u_1}{u_p} + 2\alpha u_p\Big[1 + \frac{\cos\psi}{(1+\sin\psi)}\Big] + \frac{\mathbf{m}^2\xi\eta_1 u_p^6}{64}N_2,$$

$$N_2 = S_2\sin\psi - C_2\cos\psi + f_2(\phi=\pi-\psi) = \frac{\sin^2\theta_0}{\cos\psi}\Big\{\sin 2(\phi_0+\psi)\times$$

$$\times\Big[\{144\sin^8\psi - 80\sin^6\psi - 52\sin^4\psi - 78\sin^2\psi\}\cos\psi + 39(2\psi-\pi)\sin\psi\Big]+$$

$$+\cos 2(\phi_0+\psi)\Big[152\sin^7\psi - 144\sin^9\psi + 26\sin^5\psi + 65\sin^3\psi - 99\sin\psi+$$

$$+195(\psi-\pi)\cos\psi\Big]+4[8\sin^7\psi-6\sin^5\psi-15\sin^3\psi+13\sin\psi+45(\pi-\psi)\cos\psi]\Big\}+$$

$$+\frac{16}{\cos\psi}\Big[15(\pi-\psi)\cos\psi + 7\sin\psi - 5\sin^3\psi - 2\sin^5\psi\Big]. \qquad (3.13)$$

From the second expression (21) one can obtain θ_1:

$$\theta_1 = \theta(\phi_1) = \frac{\mathbf{m}^2\xi\eta_1 u_p^6}{64}N_4\sin 2\theta_0,$$

$$N_4 = S_4\sin\psi + f_4(\phi=\pi-\psi) + f_4(\phi=0) =$$

$$= 5\sin(\phi_0+\psi)\Big\{\Big[48\sin^7\psi - 8\sin^5\psi - 10\sin^3\psi - 15\sin\psi\Big]\times$$

$$\times\cos\psi + 15(\psi-\pi)\Big\} + 48\cos(\phi_0+\psi)\Big[4\sin^6\psi - 5\sin^8\psi\Big]. \qquad (3.14)$$

This implies that the gravitational field bends the beams only in one plane.

Expression (21) shows that the beams, in which electromagnetic pulses propagate, undergo the gravitational bending (proportional to $\alpha = \gamma M/c^2$) and nonlinear electrodynamic bending (proportional to $\mathbf{m}^2\xi$). The value of the total angle of bending can be found from (22) if put $u_1 = 0$ ($r \to \infty$) and note that $\sin\psi = u_0/u_p = r_p/r_0$, where r_p is a periapsis of the beam and r_0 is radial coordinate of the point from which electromagnetic pulse was emitted. The gravitational part of bending angle in this notation takes the form:

$$\delta\varphi_{GR} = \frac{2\gamma M}{c^2 r_p}\Big[1 + \frac{\sqrt{r_0^2 - r_p^2}}{r_0 + r_p}\Big].$$

At $r_0 \to \infty$ this expression gives the well-known [41] Einsteins relation

$$\delta\varphi_{GR} = \frac{4\gamma M}{c^2 r_p},$$

when the beam starts at spatial infinity, passes by the pulsar and goes to spatial infinity. At $r_0 = r_p$ (beam starts at periapsis) this expression gives, as one would expect, half value:

$$\delta\varphi_{GR} = \frac{2\gamma M}{c^2 r_p}.$$

Nonlinear polarization of the vacuum by magnetic field at $\theta_0 \neq 0$ and $\theta_0 \neq \pi/2$ leads to a bending of the beam in the plane $\theta = \pi/2$, whose value is determined by the expression (22), as well as in the direction which allow the beam to leave given plane (23). The magnitude of these angles in the most optimistic case not more than a few arcseconds. Due to the large distances ($r \sim 10$ kps $>> r_p$) between pulsars and the Earth, the angular resolution of modern receivers does not allow us to fix the bending of beam.

4. Solution of the equations for beams of the second normal mode

For the beam on which the pulse propagates carried by the second normal mode, the expressions (3.2) take the form:

$$u(\phi) = u_p \sin(\phi + \psi) + \alpha u_p^2 \Phi_1(\phi) + \mathbf{m}^2 \xi \eta_2 u_p^7 \Phi_2(\phi),$$

$$\theta(\phi) = \frac{\pi}{2} + \alpha u_p \Phi_3(\phi) + \mathbf{m}^2 \xi \eta_2 u_p^6 \Phi_4(\phi),$$

$$x^0(\phi) = \frac{\cos\psi}{u_p \sin\psi} - \frac{\cos(\phi + \psi)}{u(\phi)} + \alpha \Phi_5(\phi) + \mathbf{m}^2 \xi \eta_2 u_p^5 \Phi_6(\phi), \tag{4.1}$$

with the same functions $\Phi_a(\phi)$, which were used for the first normal mode (3.6)-(3.7).

Integration constants for beam of the second normal mode are defined from boundary conditions: at $\phi = 0$ and $t = 0$ the beam should begin at the point $u = u_0$, $\theta = \pi/2$ and asymptotically go to spatial infinity ($r \to \infty$, $u \to 0$.) Therefore, the integration constants C_1, C_2, C_3, C_4, A_5 and A_6 will be defined by equations (3.8), (3.10)-(3.11).

For the aim of finding values of the integration constants S_1, S_2, S_3 and S_4, we should define the angle $\phi = \phi_2$, at which $u = u_1$. Substituting

$\phi = \phi_2 = \pi - \psi + \beta_2$ in the first equation of (4.1), and equating it to u_1, we obtain:

$$\beta_2 = -\frac{u_1}{u_p} + \alpha u_p\Big[2 + 2\cos\psi + S_1\sin\psi)\Big] + \frac{\mathbf{m}^2\xi\eta_2 u_p^6}{64}N_{22},$$

$$N_{22} = S_2\sin\psi - C_2\cos\psi + f_2(\phi = \pi - \psi) = S_2\sin\psi + \sin^2\theta_0\Big\{2\sin 2(\phi_0+\psi)\times$$
$$\times\Big[112\sin^8\psi - 72\sin^{10}\psi - 14\sin^6\psi + 13\sin^4\psi - 39\sin^2\psi\Big] + \cos 2(\phi_0+\psi)\times$$
$$\times\Big[\{152\sin^7\psi - 144\sin^9\psi + 26\sin^5\psi + 65\sin^3\psi\}\cos\psi + 195(\psi-\pi)\Big]+$$
$$+2\Big[16\sin^7\psi - 12\sin^5\psi - 30\sin^3\psi\Big]\cos\psi - 180(\psi-\pi)\Big\}-$$
$$-16\sin^3\psi[5 + 2\sin^2\psi]\cos\psi - 240(\psi-\pi). \qquad (4.2)$$

One can use now $\phi = \phi_2 = \pi - \psi + \beta_2$ in the second equation of (4.1). It is simple to show that

$$\theta_2 = \theta(\phi_2) = \frac{\pi}{2} + \alpha u_p S_3\sin\psi + \frac{\mathbf{m}^2\xi\eta_2 u_p^6}{64}N_{44}\sin 2\theta_0,$$

$$N_{44} = S_4\sin\psi + f_4(\phi = \pi - \psi) + f_4(\phi = 0) = S_4\sin\psi - 5\sin(\phi_0+\psi)\times$$
$$\Big\{\Big[8\sin^7\psi - 2\sin^5\psi - 5\sin^3\psi\Big]\cos\psi - 15(\psi-\pi)\Big\}+$$
$$+\cos(\phi_0+\psi)\Big[40\sin^8\psi - 38\sin^6\psi + \sin^4\psi - 3\sin^2\psi\Big]. \qquad (4.3)$$

Since at spatial infinity both beams have to get to the measuring device, the conditions which must be satisfied are the next: $\beta_1 = \beta_2,\ \theta_1 = \theta_2$. By substituting in these expressions the equations (3.13)-(3.14) and (4.2)-(4.3), we obtain:

$$S_1 = -\frac{2\cos\psi}{(1+\sin\psi)}, \quad S_3 = 0,$$

$$S_2 = \frac{u_p\eta_1}{u_0\eta_2}N_2 - \frac{u_p\sin^2\theta_0}{u_0}\Big\{2\sin 2(\phi_0+\psi)\Big[112\sin^8\psi - 72\sin^{10}\psi - 14\sin^6\psi+$$
$$+13\sin^4\psi - 39\sin^2\psi\Big] + \cos 2(\phi_0+\psi)\Big[\{152\sin^7\psi - 144\sin^9\psi + 26\sin^5\psi+$$
$$+65\sin^3\psi\}\cos\psi + 195(\psi-\pi)\Big] + 2\Big[16\sin^7\psi - 12\sin^5\psi - 30\sin^3\psi\Big]\cos\psi-$$

$$-180(\psi-\pi)\Big\} + \frac{16u_p}{u_0}\Big\{[5+2\sin^2\psi]\cos\psi\sin^3\psi - 240(\psi-\pi)\Big\},$$

$$S_4 = \frac{u_p\eta_1}{u_0\eta_2}N_4 + \frac{u_p}{u_0}\Big\{5\sin(\phi_0+\psi)\Big[[8\sin^7\psi - 2\sin^5\psi - 5\sin^3\psi]\cos\psi -$$

$$-15(\psi-\pi)\Big] - \cos(\phi_0+\psi)\Big[40\sin^8\psi - 38\sin^6\psi + \sin^4\psi - 3\sin^2\psi\Big]\Big\}.$$

Thus, all integration constants for beam of the second mode are defined.

5. Calculation of the delay time

The last paragraph we define a time interval $T_{adv} = t_1 - t_2$, which one normal mode ahead of another mode in the propagation of an electromagnetic pulse from the source to the measurement device. Using expressions (11) and (24), we obtain:

$$T_{adv} = \frac{\mathbf{m}^2\xi(\eta_1-\eta_)u_p^5}{64}\Big\{\sin^2\theta_0\Big[16\sin 2(\phi_0+\psi)[9\sin^8\psi - 4\sin^6\psi] +$$

$$+\cos 2(\phi_0+\psi)[(144\sin^7\psi + 8\sin^5\psi + 26\sin^3\psi + 39\sin\psi)\cos\psi +$$

$$+39(\pi-\psi)] - 4(8\sin^5\psi + 6\sin^3\psi + 9\sin\psi)\cos\psi + 36(\psi-\pi)\Big] -$$

$$-16[2\sin^3\psi + 3\sin\psi]\cos\psi + 48(\psi-\pi)\Big\}. \quad (5.1)$$

We estimate the numerical value of T_{adv} in the case where the magnetic field source is a neutron star with field on a surface $B \sim 10^{16}$ G (magnetar [42]). Due to the condition $\xi\mathbf{m}^2/r^6 << 1$, which must be satisfied for all points of considered beams, our calculation will be applicable only to the beams, for which the pericenter r_p exceeds ten radii of the neutron star. In this spatial region $B(r) < 10^{13}$ G and $\xi\mathbf{m}^2/r^6 \leq 0.05$. Taking into account that the radius of a typical magnetar is 10 km, from the expression (5.1) we obtain in order of magnitude the value: $T_{adv} \sim 10^{-8}$ sec.

Thus, the relatively short pulse of hard radiation arising in the magnetosphere of the pulsar or on its surface splits due to nonlinear electrodynamic birefringence into two pulses having mutually orthogonal polarization and different velocities. Faster pulse after passing through the region of strong magnetic field $B \sim 10^{13}$ G of the pulsar or magnetar will outpace slower one in 10^{-8} seconds. Therefore, the two electromagnetic pulses emitted in the same time from the same source will come to the recording device installed on the near-Earth satellite over different beams at different times $t_2 \neq t_1$. This device will first register an arrival of the front part of a more rapid pulse and the polarization of the detected radiation will be linear over time $t_2 - t_1$. After this period of time the front part of the second pulse having an orthogonal polarization will come to the recorder. Therefore, the further polarization of the total momentum will be arbitrary.

6. Conclusion

The calculations show that the effect of delay has good prospects for its experimental measurement using space instruments. In order to measure the delay of mutually perpendicularly polarized components of the gamma pulse from a neutron star one will need to create appropriate detector:

The detector must comprise an unit which measures the polarization pulse of hard radiation over its entire length. Currently, these units are under development in several research centers [43].

However, this effect can also be detected by measuring the intensity of gamma radiation from neutron stars with high resolution. The initial part of the pulse with the duration of the order of 10^{-8} sec emitted in a direction perpendicular to the magnetic axis of the neutron star will be twice smaller than the rest part of the pulse (so will be with the tail of the pulse, but it will be more difficult to identify). Unfortunately, modern detectors have an accuracy about 10^{-6} sec, therefore on the basis of the existing data to detect the effect was not possible. It is necessary to conduct exoatmospheric experiments on detection in high accuracy intensity and polarization of a hard radiation from the neutron stars.

References

[1] E. M. Berestetskiy, E. M. Lifshitz and L. P. Pitayevskiy, *Quantum Electrodynamics*, Nauka (1980).

[2] D. L. Burke, R. C. Feld, G. Horton-Smith and et al., *Positron Production in Multiphoton Light-by-Light Scattering*, *Physical Review Letters* **79** (1997) 1626.

[3] V. I. Denisov and V. A. Sokolov, *Analysis of Regularizing Properties of Nonlinear Electrodynamics in the Einstein-Born-Infeld Theory*, *Journal of Experimental and Theoretical Physics* **113** (2011) 926.

[4] Jin Young Kim, *Bending of electromagnetic wave in an ultra-strong magnetic field*, *Journal of Cosmology and Astroparticle Physics* **10** (2012) 056.

[5] P. A. Vshivtseva, V. I. Denisov and I. P. Denisova, *Nonlinear electrodynamic effect of frequency doubling in the field of a magnetic dipole*, *Doklady Physics* **47** (2002) 798.

[6] Jin Young Kim and Taekoon Lee, *Light bending in radiation background*, *Journal of Cosmology and Astroparticle Physics* **01** (2014) 002.

[7] V. I. Denisov, N. V. Kravtsov and I. V. Krivchenkov, *A nonlinear electrodynamical shift of spectral lines in the hydrogen atom and hydrogen-like ions*, *Optics and Spectroscopy* **100** (2006) 641.

[8] Remo Ruffini, Yuan-Bin Wu, and She-Sheng Xue, *Einstein-Euler-Heisenberg theory and charged black holes*, *Physical Review* **88** (2013) 085004.

[9] V. I. Denisov, I. P. Denisova and I. V. Krivchenkov,, *Hamilton-Jacobi equation for fermions interacting nonminimally with electromagnetic field*, *Doklady Physics* **48** (2003) 325.

[10] V. I. Denisov, N. V. Kravtsov and I. V. Krivchenkov,, *On the possibility of observing polarisation of vacuum in a magnetic field*, *Quantum Electronics* **33** (2003) 938.

[11] V.A. Sokolov, *Electromagnetic wave interactions in a strong permanent magnetic field*, *Moscow University Physics Bulletin* **64** (2009) 349.

[12] V. I. Denisov and I. P. Denisova, *Interaction effect of plane electromagnetic waves in the Born-Infeld nonlinear electrodynamics*, *Theoretical and Mathematical Physics* **129** (2001) 1421.

[13] J. Han and S. Kim, *Maxwell-Chern-Simons vortices on compact surfaces: Nonequivalence of the first and the second order equations*, *J. Math. Phys.* **54** (2013) 023505.

[14] V. I. Denisov and I. P. Denisova, *Interaction of intense laser radiation with weak electromagnetic waves in an evacuated section of a ring laser*, *Optics and Spectroscopy* **90** (2001) 928.

[15] Gert Brodin, Mattias Marklund, and Lennart Stenflo, *Proposal for Detection of QED Vacuum Nonlinearities in Maxwell's Equations by the Use of Waveguides*, *Physical Review Letters* **87** (2001) 171801.

[16] V. I. Denisov and I. P. Denisova, *Verifiable post-Maxwellian effect of the nonlinear electrodynamics in vacuum*, *Optics and Spectroscopy* **90** (2001) 282.

[17] Selym Villalba-Chavez, *Photon magnetic moment and vacuum magnetization in an asymptotically large magnetic field*, *Physical Review D* **81** (2010) 105019.

[18] V. I. Denisov, *Nonlinear effect of quantum electrodynamics for experiments with a ring laser*, *Journal of Optics: a-Pure and Applied Optics,* **2** (2000) 372.

[19] V. I. Denisov, I. P. Denisova and S. I. Svertilov, *Nonlinear electrodynamic delay of electromagnetic signals in a Coulomb field*, *Theoretical and Mathematical Physics* **135** (2003) 720.

[20] Novick, R.; Weisskopf, M. C.; Angel, J. R. P.; Sutherland, P. G. The effect of vacuum birefringence on the polarization of X-ray binaries and pulsars Astrophysical Journal, vol. 215, , p. L117-L120 (1977)

[21] Gnedin, Yu. N.; Silantiev, N.A. X-ray polarization magnetized accretion disk Soviet Astronomy Letters 1977, 3, 258.

[22] Gnedin, Yu. N.; Pavlov, G. G.; Shibanov, Yu. A. The effect of vacuum birefringence in a magnetic field on the polarization and beaming of X-ray pulsars Soviet Astronomy Letters, vol. 4, p. 117-119. (1978)

[23] V. A. Sokolov, *Generation of Second and Third Harmonics by a Rotating Pulsar in Parametrized Post-Maxwellian Electrodynamics*, *Moscow University Physics Bulletin* **63** (2008) 279.

[24] V. I. Denisov and S. I. Svertilov, *Vacuum nonlinear electrodynamics curvature of photon trajectories in pulsars and magnetars*, *Astronomy & Astrophysics* **399** (2003) L39.

[25] Jin Young Kim and Taekoon Lee, *Light bending by nonlinear electrodynamics under strong electric and magnetic field*, *Journal of Cosmology and Astroparticle Physics* **11** (2011) 017.

[26] S. L. Adler, *Photon splitting and photon dispersion in a strong magnetic field*, *Annals of Physics* **67** (1971) 599.

[27] V. I. Denisov, I. P. Denisova and S. I. Svertilov, *Nonlinear electrodynamic effect of ray bending in the magnetic-dipole field*, *Doklady Physics* **46** (2001) 705.

[28] V. I. Denisov, I. P. Denisova and I. V. Krivchenkov, *Path equation for photons moving in the magnetic meridian plane of a dipole magnetic field and governed by the laws of nonlinear electrodynamics in a vacuum*, *Doklady Mathematics* **67** (2003) 90.

[29] V. I. Denisov, I. P. Denisova and S. I. Svertilov, *Nonlinear electromagnetic delay of electromagnetic signals propagating in the magnetic meridian plane of pulsars and magnetars*, *Theoretical and Mathematical Physics* **140** (2004) 1001.

[30] V. I. Denisov, Krivchenkov, I. V. and I. P. Denisova, *Nonlinear electrodynamic lag of electromagnetic signals in a magnetic dipole field*, *Journal of Experimental and Theoretical Physics* **95** (2002) 194.

[31] V. I. Denisov, *Investigation of the effective space-time of the vacuum nonlinear electrodynamics in a magnetic dipole field*, *Theoretical and Mathematical Physics* **132** (2002) 1071.

[32] V. A. Fock, *Theory of Space, Time and Gravitation*, Pergamon (1961).

[33] V. I. Denisov and I. P. Denisova, *The eikonal equation in parameterized nonlinear electrodynamics of vacuum*, *Doklady Physics* **46** (2001) 377.

[34] S. Chandrasekhar, *The mathematical theory of black hole*, Oxford University Press (1983).

[35] V. I. Denisov and M. I. Denisov, *Verification of Einstein's principle of equivalence using a laser gyroscope in terrestrial conditions*, *Physical Review D* **60** (1999) 047301.

[36] R. Courant, *Partial differential equations*, London (1962).

[37] P. A. Vshivtseva and M. M. Denisov, *Mathematical Modelling of electromagnetic wave propagation in nonlinear electrodynamics*, *Computational Mathematics and Mathematical Physics* **49** (2009) 2092.

[38] M. S. Strickman, A. K. Harding and O. C. De Jager, *An RXTE Observation of the Vela Pulsar: Filling in the X-Ray Gap*, *Astrophysical Journal* **524** (1999) 373.

[39] Ch. Harper, *Introduction to Mathematical Physics*, Prentice-Hall (1976).

[40] V. M. Kaspi, J. R. Lackey, J. Mattox, R. N. Manchester, M. Bailes and R. Pace, *High-energy gamma-ray observations of two young, energetic radio pulsars*, *Astrophysical Journal* **528** (2000) 445.

[41] A. Einstein Erklarung der Perihelbewegung der Merkur aus der allgemienen Relativitatstheorie. Sitzungsber. Preuss. Akad. Wiss., volume 47, 2, (1915) pages 831-839

[42] A. Colaiuda, V. Ferrari, L. Gualtieri1 and J. A. Pons, *Relativistic models of magnetars: structure and deformations*, *Mon. Not. R. Astron. Soc.* **385** (2008) 2080.

[43] X-ray polarimetry: a new window in astrophysics. Edited by Ronaldo Bellazzini, Enrico Costa, Giorgio Matt and Gianpiero Tagliaferri,.Cambridge University Press 2010