Electrodynamic Properties of Dense Semiclassical Plasmas

A. K. Baidualiyeva, K. N. Dzhumagulova, T. S. Ramazanov, and A. Bakirzhankyzy

***Abstract*— In this paper, the dielectric function of dense semiclassical collisionless plasmas is investigated on the basis of the interaction potential, which takes into account the effects of diffraction in a wide range of temperatures and densities. The dielectric function is analytically and numerically investigated in approximation of high frequencies. All the obtained results are in good agreement.**

***Index Terms*— Plasma waves.**

I. Introduction

It is well known that the dielectric function plays a key role in the description of the electrodynamic properties of plasmas. Using it, one can describe the spectrum of the plasma waves, optical properties, as well as many other phenomena [1]–[3]. In dense plasmas, the influence of the many-body effects and quantum mechanical effects increases. In this case, the dielectric function can significantly differ from the dielectric function of rarefied plasmas. To adequately determine the dielectric function, it is necessary to know the interaction potential of the plasma particles. Development of the particle interaction models and study of the properties of strongly coupled dense plasmas on their basis are of great fundamental and practical interest [4]–[9]. To take into account quantum mechanical effects in the interaction potential, a special method was developed. It consists of the comparison of the classical Boltzmann factor and the quantum mechanical Slater sum. This approach was first described in [10]. The Deutsch potential [8], [9], which correctly considers the diffraction effect only at high temperatures, has the following form:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{r}\left(1 - e^{-\frac{r}{\lambda_{\alpha\beta}}}\right). \tag{1}$$

Here, $\lambda_{\alpha\beta} = \hbar/(2\pi m_{\alpha\beta} k_B T)^{1/2}$ is the de Broglie thermal wavelength and $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass of α- and β-interacted particles. In this paper, the following dimensionless parameters are used: $\Gamma = Z_\alpha Z_\beta e^2/(a\, k_B T)$ is the coupling parameter [the average distance between particles is $a = (3/4\pi n)^{1/3}$; $n = n_e + n_i$ is the numerical density of the electrons and ions; T is the plasma temperature; and k_B is the Boltzmann constant] and $r_s = a/a_B$ is the density parameter ($a_B = \hbar^2/m_e e^2$ is the Bohr radius).

In [11], the interaction micropotential of the dense semiclassical plasma was obtained on the basis of the method described in [10] with help of interpolation of the numerical results in a wide region of temperatures and densities

$$\hat{O}_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{r}\left(1 - \text{than}\left(\sqrt{2}\frac{\lambda_{\alpha\beta}^2}{a^2 + br^2}\right) e^{-\text{than}\left(\sqrt{2}\frac{\lambda_{\alpha\beta}^2}{a^2+br^2}\right)}\right) \times (1 - e^{-r/\lambda_{\alpha\beta}}), \quad b = 0.033 \tag{2}$$

where a is the average distance between particles. This micropotential (2) takes into account the quantum diffraction effect in a wide region of temperatures and densities.

Dielectric function $\varepsilon(\omega, k)$ is defined as the value characterizing the magnitude of charge screening in plasmas. Dielectric function of collisionless plasmas in high-frequency limit can be presented by the following expression [1]:

$$\varepsilon(k, \omega) = 1 - \chi_e^0(k, \omega)\tilde{\varphi}_{ee}(k) \tag{3}$$

where $\tilde{\varphi}_{ee}(k)$ is the Fourier transform of the interaction micropotential between the electrons, and the response function of the system of noninteracting particles is

$$\chi_e^0(k, \omega) = -\frac{n_e}{k_B T} W\left(\frac{\omega}{k \mathrm{v}_{\mathrm{Te}}}\right) \tag{4}$$

where v_{Te} is the thermal velocity of the electrons and k is a wave vector

$$W(z) = 1 - z\exp(-z^2/2) \cdot \int_0^z \exp(y^2/2)dy + i\sqrt{\frac{\pi}{2}} z \exp(-z^2/2). \tag{5}$$

Function $W(z)$ in the asymptotic expansion at the high-frequency approximation $\omega/k v_{\mathrm{Te}} \gg 1$ is

$$W(z) = iz\sqrt{\frac{\pi}{2}}\exp\left(-\frac{z^2}{2}\right) - \frac{1}{z^2} - \frac{3}{z^4} - \cdots \tag{6}$$

II. Tasks and Results

In this paper, the dielectric function of dense semiclassical plasmas is obtained on the basis of the potential (2). For obtaining an analytical expression for the dielectric function, the exponents and tangent in the potential (2) were expanded, and only the first term, giving the main contribution, was taken

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The authors are with the Faculty of Physics and Technology, Institute of Experimental and Theoretical Physics, Al-Farabi Kazakh National University, Almaty 050040, Kazakhstan (e-mail: baidualieva.aktoty55@gmail.com; dzhumagulova.karlygash@gmail.com; ramazan@physics.kz; aiko9119@mail.ru).

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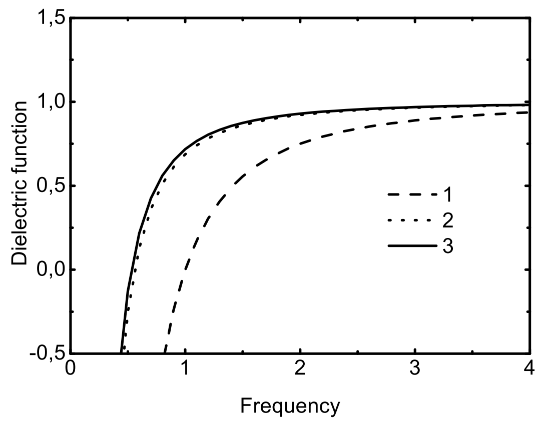

Fig. 1. Real part of the dielectric function obtained on the basis of 1—formula (8), 2—the Deutsch potential, and 3—formula (7). $\Gamma = 0.5$, $ka = 0.1$, and $r_s = 5$.

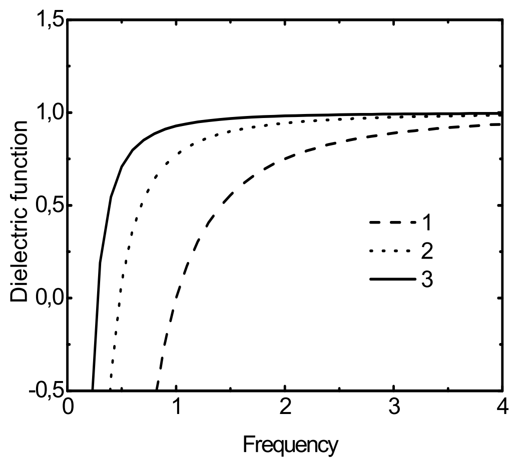

Fig. 2. Real part of the dielectric function obtained on the basis of 1—formula (8), 2—the Deutsch potential, and 3—formula (7). $\Gamma = 1$, $ka = 0.1$, and $r_s = 5$.

into account. The Fourier transform of such simplified form of the interaction potential (2) was deduced analytically and then we obtained the following expression for the real part of the dielectric function for collisionless plasmas in high-frequency limit within asymptotic approximation:

$$\mathrm{Re}(\varepsilon(k^*, \omega^*)) = 1 - \frac{(k^*)^2}{(\omega^*)^2} \cdot \left(\frac{1}{(k^*)^2 + \frac{2\Gamma}{\pi r_s}} + \frac{\pi \frac{2\Gamma}{\pi r_s}}{k^* \sqrt{2b}} - \frac{\pi \left(\frac{2\Gamma}{\pi r_s}\right)^2}{k^* 2\sqrt{b}} \times \left(1 + \frac{k^*}{\sqrt{b}}\right) \exp\left(-\frac{k^*}{\sqrt{b}}\right) \right). \quad (7)$$

Here, dimensionless wave vector and frequency are $\omega^* = \omega/\omega_p$ and $k^* = ka$, respectively, where $\omega_p = (4\pi n_e e^2/m_e)^{1/2}$ is the electron Langmuir frequency. The real part of the dielectric function within the Coulomb potential in this approach is presented by the following expression:

$$\mathrm{Re}(\varepsilon(\omega^*)) = 1 - \frac{1}{(\omega^*)^2}. \quad (8)$$

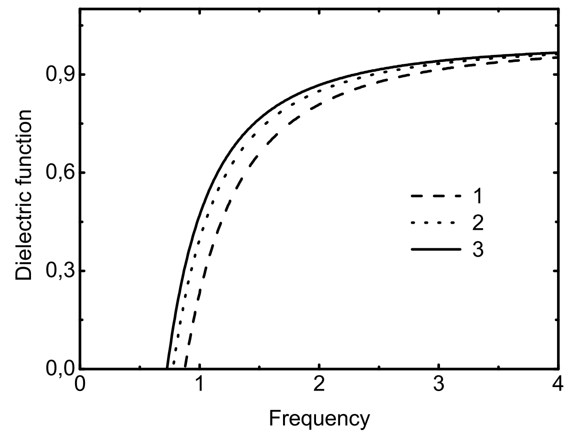

Fig. 3. Numerical calculation of the dielectric function in the asymptotic approximation obtained on the basis of 1—formula (8), 2—the Deutsch potential, and 3—the potential (2). $\Gamma = 0.1$, $ka = 0.1$, and $r_s = 5$.

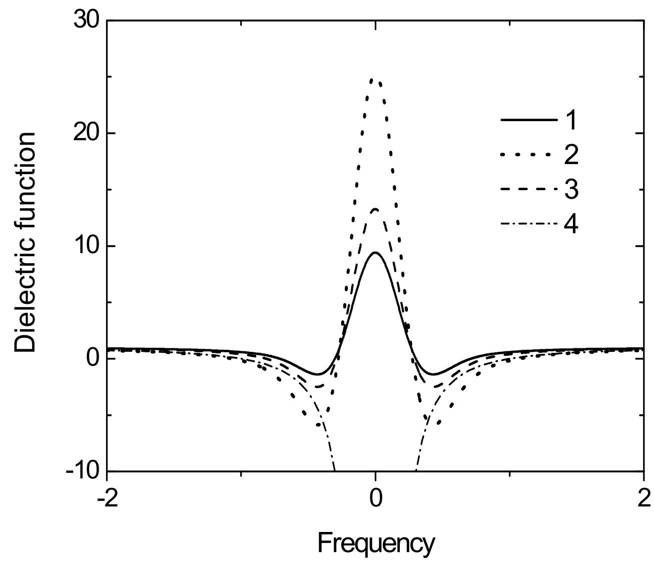

Fig. 4. Dielectric function obtained without asymptotic expansion on the basis of 1—the potential (2), 2—the Coulomb potential, 3—the Deutsch potential, and 4—formula (8). $\Gamma = 5$, $ka = 0.78$, and $r_s = 1$.

The real parts of the dielectric function obtained by formula (7), obtained for the Coulomb potential by formula (8), and obtained for the Deutsch potential are shown in Figs. 1 and 2. One can see that the real part of the dielectric function obtained on the basis of the potential (2) [expression (7)] lies above the other curves and tends to the data obtained on the basis of the Deutsch potential on decreasing the coupling parameter.

For more precise estimation of the dielectric function, we used again (3), (4), and (6), but instead of an analytical expression for the Fourier transform of the potential (2), we used a numerical method for its calculation. As a result, we received data that qualitatively agree with formula (7) (Fig. 3).

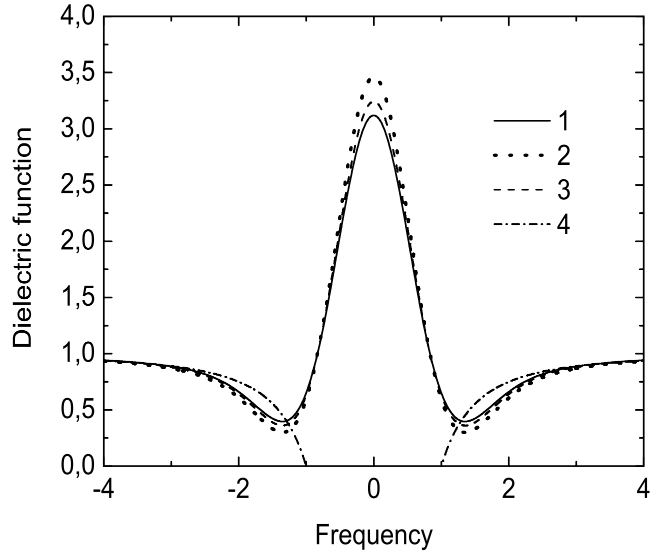

Fig. 5. Dielectric function obtained without asymptotic expansion on the basis of 1—the potential (2), 2—the Coulomb potential, 3—the Deutsch potential, and 4—formula (8). $\Gamma = 0.5$, $ka = 0.78$, and $r_s = 1$.

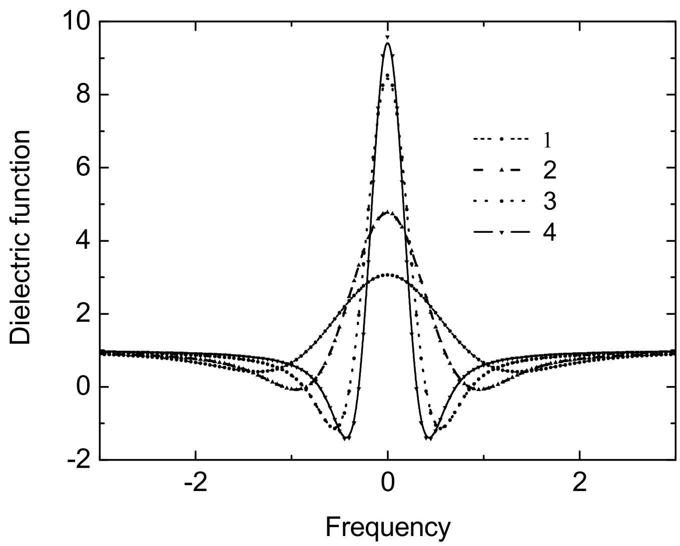

Fig. 6. Dielectric function obtained without asymptotic expansion on the basis of the potential (2) at different coupling parameters, $ka = 0.78$ and $r_s = 1$. 1: $\Gamma = 0.5$. 2: $\Gamma = 1$. 3: $\Gamma = 3$. 4: $\Gamma = 5$.

In the third approach, we obtained the dielectric function on the basis of the numerically calculated $W(z)$ (5). The obtained results are presented in Figs. 4–6.

In Figs. 4 and 5, one can see that the curves obtained on the basis of the Deutsch potential and potential (2) are close to each other and differ from the result obtained on the basis of the Coulomb potential on increasing the coupling parameter. However, the result obtained on the basis of the Deutsch potential lie between results on the basis of the Coulomb potential and potential (2). In Fig. 6, the dielectric functions obtained on the basis of the potential (2) at different values of the coupling parameter are shown.

Based on all the obtained results, one can conclude that taking into account the diffraction effect in a wide region of temperature and densities can lead to perceptible change in the dielectric function.

REFERENCES

[1] S. Ichimaru, "Strongly coupled plasmas: High-density classical plasmas and degenerate electron liquids," *Rev. Modern Phys.*, vol. 54, no. 4, pp. 1017–1059, 1982.

[2] Y.-D. Jung and W.-P. Hong, "Electron spin-interaction effects on the ponderomotive magnetization and radiation in degenerate quantum plasmas," *Phys. Plasmas*, vol. 20, no. 8, p. 084502, 2013.

[3] W. R. Johnson, C. Guet, and G. F. Bertsch, "Optical properties of plasmas based on an average-atom model," *J. Quant. Spectrosc. Radiat. Transf.*, vol. 99, nos. 1–3, pp. 327–340, 2006.

[4] T. S. Ramazanov and K. N. Dzhumagulova, "Effective screened potentials of strongly coupled semiclassical plasma," *Phys. Plasmas*, vol. 9, no. 9, p. 3758, 2002.

[5] C. Deutsch, "Nodal expansion in a real matter plasma," *Phys. Lett. A*, vol. 60, no. 4, pp. 317–318, 1977.

[6] T. S. Ramazanov, K. N. Dzhumagulova, Y. A. Omarbakiyeva, and G. Röpke, "Effective polarization interaction potentials of the partially ionized dense plasma," *J. Phys. A, Math. General*, vol. 39, no. 17, p. 4369, 2006.

[7] Y.-D. Jung and D. Kato, "Quantum effects on the entanglement fidelity in elastic scatterings in strongly coupled semiclassical plasmas," *Phys. Plasmas*, vol. 15, no. 10, p. 104503, 2008.

[8] C. Deutsch and M. M. Gombert, "Diffraction corrections to the equilibrium properties of the classical electron gas. Pair correlation function," *J. Math. Phys.*, vol. 17, no. 7, p. 1077, 1976.

[9] C. Deutsch, Y. Furutani, and M. M. Gombert, "Nodal expansions for strongly coupled classical plasmas," *Phys. Rep.*, vol. 69, no. 2, pp. 85–193, 1981.

[10] A. A. Barker, "Effective potentials between the components of a hydrogeneous plasma," *J. Chem. Phys.*, vol. 55, no. 4, p. 1751, 1971.

[11] Z. A. Moldabekov, T. S. Ramazanov, and K. N. Dzhumagulova, "Pair interaction potential of particles for two-component plasma," *Contrib. Plasma Phys.*, vol. 52, no. 3, pp. 207–210, 2012.

A. K. Baidualiyeva was born in Kazakhstan in 1992. She received the master's degree in physics from the Faculty of Physics and Technology, Al-Farabi Kazakh National University, Almaty, Kazakhstan, in 2013, with a specialization in physics, and the master's degree in physics from Al-Farabi Kazakh National University.

She participated in international conferences and has authored five publications. Her current research interests include the study of the properties of the non-ideal plasma, in particular, its optical properties.

Ms. Baidualiyeva received the Scholarship of Academician F. Baymbetov.

K. N. Dzhumagulova was born in Zhambyl, Kazakhstan, in 1971. She received the Doctor of Physical and Mathematical Sciences degree from the Physical Faculty, Al-Farabi Kazakh National University, Almaty, Kazakhstan, in 1993, with a specialization in physics. In 1997, she defended her thesis on the thermodynamic properties and the distribution of a microfield of a non-ideal plasma on the specialty Physics and Chemistry of Plasma. In 2008, she defended her Ph.D. thesis on Structural, Thermodynamic and Transport Properties of Complex Plasma on the Basis of the Pseudopotential Models on the specialty Plasma Physics.

She is currently the Head of the scientific projects funded by the Ministry of Education and Sciences of Kazakhstan. She teaches courses for bachelor's, master's, and Ph.D. doctors at a high pedagogical level. She is a Highly Cited Scientist from Kazakhstan. She has authored over 150 scientific articles in national and international journals, including about 40 in refereed journals with non-zero impact factor. She has patents of the Republic of Kazakhstan. Her current research interests include the study of the properties of the non-ideal plasma of complex composition.

Prof. Dzhumagulova received the State Scholarship for talented young scientists from 2002 to 2005, the Grant of the German Academic Exchange Service (Training in Germany) in 2002, and the Best University Teacher Grants in 2007 and 2013.

T. S. Ramazanov was born in Atyrau, Kazakhstan, in 1961. He received the Doctor of Physical and Mathematical Sciences degree from the Physical Faculty, Al-Farabi Kazakh National University, Almaty, Kazakhstan, named after S. M. Kirov.

He is currently a Doctor of physical and mathematical sciences and a Professor. He supervised one D.Sc. candidate, 13 candidates of sciences, and four Ph.D. doctors. He has authored over 350 scientific publications, three of which are in the peer-reviewed scientific journals with the high impact-factor. He made more than 120 scientific reports in prestigious international conferences. He has co-authored three monographs, 15 methodological papers, and a book entitled *Electricity and Magnetism* in Kazakh language.

Prof. Ramazanov was a nominee of the State Grant of Ministry of Education and Science of R. K. The Best Teacher at High School in 2006 and 2010, and twice the State Scientific Grant for the scientist and experts who have brought in the outstanding contribution to the development of science and engineering from 2002 to 2006.

A. Bakirzhankyzy was born in Kazakhstan in 1992. She received the master's degree in physics from the Faculty of Physics and Technology, Al-Farabi Kazakh National University, Almaty, Kazakhstan, in 2013, with a specialization in physics, and the master's degree in physics from Al-Farabi Kazakh National University.

She participated in international conferences and has authored six publications.