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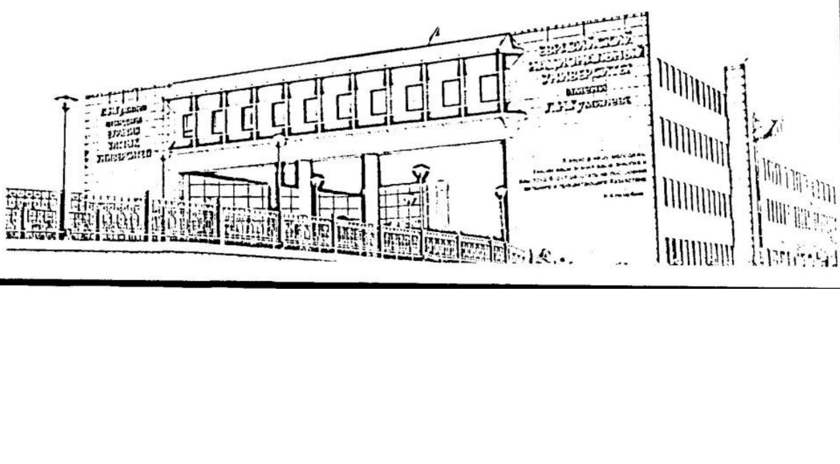

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БӨЛІМ

МАЗМҰНЫ СОДЕРЖАНИЕ

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Kemelbekova G.M., Esmakhanova K.R., Tapeeva S.K., Tungushbaeva D.I. and Myrzakulov R.

Darboux transformation and one-soliton solution of the (2+1)-dimensional modified Korteweg-de Vries equation

(Eurasian International Center for Theoretical Physics and Department of General and Theoretical Physics, Eurasian National University L.N. Gumiliyov)

Darboux transformation, a comprehensive approach to construct the explicit solutions of the nonlinear evolutionary equation, is applied to construct the solution of the modified Korteweg-de Vries (KdV) equation. In this paper, we construct a Darboux transformation of the (2+1)-dimensional modified KdV equation which is integrable by the Inverse Scattering Method. Using this transformation, the one-soliton solution is obtained from the "seed"solutions.

Keywords: Darboux transformation, soliton solution, Lax presentation, modified Kortewedge-de Vries equation

The (2+1)-dimensional modified KdV equation

The (2+1)-dimensional modified Korteweg-de Vries equation (KdV) reads as [28]

$$\begin{aligned} iq_t + iq_{xxy} - vq + i(wq)_x - 2ip &= 0, & (1)\\ v_x - 2i\delta(q^*_{xy}q - q^*q_{xy}) &= 0, & (2)\\ w_x - 2\delta(|q|^2)_y &= 0, & (3)\\ p_x - 2i\mu p - 2\eta q &= 0, & (4)\\ \eta_x + \delta(q^*p + p^*q) &= 0, & (5)\end{aligned}$$

where q, p are complex function, v, w, η are real functions, δ is real constant and λ, μ are spectral parameters. It is the (2+1)-dimensional modified KdV equation. This set of equations (2.1)-(2.5) is integrable by IST. The corresponding linear eigenvalue problem can take the form

$$\begin{aligned} \Psi_x &= A\Psi, & (6)\\ \Psi_t &= 4\lambda^2\Psi_y + B\Psi, & (7)\end{aligned}$$

where

$$\Psi = (\Psi_1, \Psi_2)^T, \tag{8}$$

$$\begin{aligned} A &= -i\lambda\sigma_3 + A_0, & (9)\\ B &= \lambda B_1 + B_0 + \frac{i}{\lambda+\mu}B_{-1}. & (10)\end{aligned}$$

Here σ_3, A_0, B_1, B_0, B_{-1} are 2×2 matrices have form

$$\begin{aligned} \sigma_3 &= \begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix}, \quad A_0 = \begin{pmatrix} 0 & q\\ -r & 0\end{pmatrix}, & (11)\\ B_1 &= \begin{pmatrix} i\mu & q\\ -r & 0\end{pmatrix}, & (12)\\ B_0 &= \begin{pmatrix} -\frac{i}{2}v & -q_{xy} - wq\\ r_{xy} + wr & \frac{i}{2}v\end{pmatrix}, & (13)\\ B_{-1} &= \begin{pmatrix} \eta & -p\\ -k & -\eta\end{pmatrix}. & (14)\end{aligned}$$

and $r = \delta q^*$, $k = \delta p^*$, where $\delta = \pm 1$. The integrability condition of (2.6)-(2.7) is still $\Psi_{xt} = \Psi_{tx}$:

$$A_t - 4\lambda^2 A_y - B_x + [A, B] = 0. \tag{15}$$

DT for the (2+1)-dimensional modified KdV equation

It is well-known that the DT has been proved to be an efficient way to find the exact solutions like solitons, dromions, positons, breathers, rogue wave solutions for integrable equations in 1+1 and 2+1 dimensions. In this section, considering the particularity of the Lax representation, we construct the DT of the (2+1)-dimensional modified KdV equation (2.1)-(2.5). Furthermore, we will find some solutions of the (2+1)-dimensional modified KdV equation using its DT.

One-fold DT

We consider the following transformation about linear function $\Psi^{[1]}(x, y, t, \lambda)$

$$\Psi^{[1]}(x, y, t, \lambda) = T(x, y, t, \lambda)\Psi(x, y, t, \lambda), \tag{16}$$

where $T(x, y, t, \lambda)$ is called a Darboux matrix

$$T(x, y, t, \lambda) = \lambda I - M(x, y, t, \lambda), \tag{17}$$

here I, M are 2×2 matrices have form

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}. \end{pmatrix}. \tag{18}$$

The new function $\Psi^{[1]}$ satisfies the next system

$$\Psi^{[1]}_x = A^{[1]}\Psi^{[1]}, \tag{19}$$

$$\Psi^{[1]}_t = 4\lambda^2\Psi^{[1]}_y + B^{[1]}\Psi^{[1]}, \tag{20}$$

where $A^{[1]}$ and $B^{[1]}$ depend on $q^{[1]}$, $v^{[1]}$, $p^{[1]}$, $\eta^{[1]}$, $w^{[1]}$ and λ. Then the matrix T can be proven to satisfy following equations

$$T_x + TA = A^{[1]}T, \tag{21}$$

$$T_t + TB = 4\lambda^2 T_y + B^{[1]}T. \tag{22}$$

The correspondence between $q^{[1]}$, $v^{[1]}$, $p^{[1]}$, $\eta^{[1]}$, $w^{[1]}$ and q, v, p, η, w can be got by (3.6)-(3.7) Collecting the different power coefficients of λ^i we obtain from (3.6) the following set of identities

$$\lambda^2 : \ iI\sigma_3 = i\sigma_3 I, \tag{23}$$

$$\lambda^1 : \ A^{[1]}_0 = A_0 + i[M, \sigma_3], \tag{24}$$

$$\lambda^0 : \ M_x = A^{[1]}_0 M - MA_0. \tag{25}$$

From the above identities, after simplifications (3.9) we get

$$q^{[1]} = q - 2im_{12}, \quad {q^*}^{[1]} = q^* - 2im_{21}. \tag{26}$$

After this system we derive $m_{21} = -m^*_{12}$ for attractive interaction case that is if $\delta = +1$. From (3.12) we take $m_{22} = m^*_{11}$. Then comparing different power coefficients of λ^i of the two sides of the equation (3.7) gives us

$$\lambda^2 : \ 4M_y = B^{[1]}_1 - B_1, \tag{27}$$

$$\lambda^1 : \ B^{[1]}_0 = B_0 - MB_1 + B^{[1]}_1 M, \tag{28}$$

$$\lambda^0 : \ -M_t = iB^{[1]}_{-1} - B^{[1]}_0 M - iB_{-1} + MB_0, \tag{29}$$

$$(\lambda + \mu)^{-1} : \ -i\mu B^{[1]}_{-1} - iB^{[1]}_{-1}M + i\mu B_{-1} + iMB_{-1} = 0. \tag{30}$$

Subsequently we get the DT

$$B_0^{[1]} = B_0 - MB_1 + (B_1 + 4M_y)M, \tag{31}$$

$$4M_y = B_1^{[1]} - B_1, \tag{32}$$

$$B_{-1}^{[1]} = (M + \mu I)B_{-1}(M + \mu I)^{-1}. \tag{33}$$

Then one-fold Darboux transformation of Kortweg-de Vries equation is expressed as

$$v^{[1]} = v + 4(m_{12}q_y^* + m_{12}^* q_y + 2im_{11}m_{11y} - 2im_{12}^* m_{12y}), \tag{34}$$

$$w^{[1]} = w - 4im_{11y} = w + 4im_{22y}. \tag{35}$$

So the matix M has the form

$$M = \begin{pmatrix} m_{11} & m_{12} \\ -m_{12}^* & m_{11}^* \end{pmatrix}, \quad M^{-1} = \frac{1}{|m_{11}|^2 + |m_{12}|^2} \begin{pmatrix} m_{11}^* & -m_{12} \\ m_{12}^* & m_{11} \end{pmatrix}, \tag{35}$$

$$M + \mu I = \begin{pmatrix} m_{11} + \mu & m_{12} \\ -m_{12}^* & \mu + m_{11}^* \end{pmatrix}, \quad (M + \mu I)^{-1} = \frac{1}{\square} \begin{pmatrix} m_{11}^* + \mu & -m_{12} \\ m_{12}^* & \mu + m_{11} \end{pmatrix}. \tag{35}$$

Here

$$\square = det(M + \mu I) = \mu^2 + \mu(m_{11} + m_{11}^*) + |m_{11}|^2 + |m_{12}|^2. \tag{35}$$

The equation (3.19)) gives

$$\eta^{[1]} = \frac{(|\mu + m_{11}|^2 - |m_{12}|^2)\eta - pm_{12}^*(\mu + m_{11}) - p^* m_{12}(\mu + m_{11}^*)}{\square}, \tag{36}$$

$$p^{[1]} = \frac{p(\mu + m_{11})^2 - p^* m_{12}^2 + 2\eta m_{12}(\mu + m_{11})}{\square}, \tag{37}$$

$$p^{*[1]} = \frac{p^*(\mu + m_{11}^*)^2 - pm_{12}^{*2} + 2\eta m_{12}^*(\mu + m_{11}^*)}{\square}. \tag{38}$$

Suppose M matrix

$$M = H\Lambda H^{-1}, \tag{39}$$

where

$$H = \begin{pmatrix} \psi_1(\lambda_1; t, x, y) & \psi_1(\lambda_2; t, x, y) \\ \psi_2(\lambda_1; t, x, y) & \psi_2(\lambda_2; t, x, y) \end{pmatrix}. \tag{40}$$

We consider

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{41}$$

and $det\ H \neq 0$, where λ_1 and λ_2 are complex constants. Using equation (21), it is easy to give

$$H_x = -i\sigma_3 H\Lambda + A_0 H. \tag{42}$$

While from (22)

$$H_t = 4H_y\Lambda^2 + B_1 H\Lambda + B_0 H + B_{-1} H\Sigma, \tag{43}$$

where

$$\Sigma = \begin{pmatrix} \frac{i}{\lambda_1 + \mu} & 0 \\ 0 & \frac{i}{\lambda_2 + \mu} \end{pmatrix}. \tag{44}$$

To satisfy the restriction of M and $B_{-1}^{[1]}$ as mentioned above, we first note that if $\delta = 1$ then

$$\Psi^+ = \Psi^{-1}, \quad A_0^+ = -A_0, \tag{45}$$

$$\lambda_2 = \lambda_1^*, \quad H = \begin{pmatrix} \psi_1(\lambda_1;t,x,y) & -\psi_2^*(\lambda_1;t,x,y) \\ \psi_2(\lambda_1;t,x,y) & \psi_1^*(\lambda_1;t,x,y) \end{pmatrix}, \tag{46}$$

$$H^{-1} = \frac{1}{\Delta}\begin{pmatrix} \psi_1^*(\lambda_1;t,x,y) & \psi_2^*(\lambda_1;t,x,y) \\ -\psi_2(\lambda_1;t,x,y) & \psi_1(\lambda_1;t,x,y) \end{pmatrix}. \tag{47}$$

So for the matrix M we have

$$M = \frac{1}{\Delta}\begin{pmatrix} \lambda_1|\psi_1|^2 + \lambda_2|\psi_2|^2 & (\lambda_1 - \lambda_2)\psi_1\psi_2^* \\ (\lambda_1 - \lambda_2)\psi_1^*\psi_2 & \lambda_1|\psi_2|^2 + \lambda_2|\psi_1|^2) \end{pmatrix}, \tag{48}$$

where

$$\Delta = |\psi_1|^2 + |\psi_2|^2. \tag{49}$$

Here we mention that $m_{22} = m_{11}^*$ and $m_{21} = -m_{12}^*$ if $\lambda_2 = \lambda_1^*$. So far, we found construction of DT of the (2+1)-dimensional modified KdV equation:

$$q^{[1]} = q - 2im_{12}, \tag{50}$$

$$v^{[1]} = v + 4(m_{12}q_y^* + m_{12}^*q_y + 2im_{11}m_{11y} - 2im_{12}^*m_{12y}), \tag{51}$$

$$w^{[1]} = w - 4im_{11y} = w + 4im_{11y}^*, \tag{52}$$

$$\eta^{[1]} = \frac{(|\mu + m_{11}|^2 - |m_{12}|^2)\eta - pm_{12}^*(\mu + m_{11}) - p^*m_{12}(\mu + m_{11}^*)}{\square}, \tag{53}$$

$$p^{[1]} = \frac{p(\mu + m_{11})^2 - p^*m_{12}^2 + 2\eta m_{12}(\mu + m_{11})}{\square}. \tag{54}$$

One-soliton solutions

As far as we got Darboux transformation, our next aim is to construct the one-soliton solution of KdV equation. In order to obtain soliton solution, the seed solution we take $q = 0$, $v = 0$, $p = 0$, $w = 0$, $\eta = 1$. Let $\lambda_1 = a + bi$. Hence the corresponding associated linear system takes the form

$$\Psi_{1x} = -i\lambda\Psi_1, \tag{55}$$

$$\Psi_{2x} = i\lambda\Psi_2, \tag{56}$$

$$\Psi_{1t} = 4\lambda^2\Psi_{1y} + \frac{i}{\lambda + \mu}\Psi_1, \tag{57}$$

$$\Psi_{2t} = 4\lambda^2\Psi_{2y} - \frac{i}{\lambda + \mu}\Psi_2. \tag{58}$$

From the above system, we take take eigenfunctions in the form

$$\Psi_1 = e^{-i\lambda_1 x + i\mu_1 y + i(4\lambda_1^2\mu_1 + \frac{1}{\lambda_1 + \mu})t + \delta_1 + i\delta_2}, \tag{59}$$

$$\Psi_2 = e^{i\lambda_1 x - i\mu_1 y - i(4\lambda_1^2\mu_1 + \frac{1}{\lambda_1 + \mu})t - \delta_1 - i\delta_2 + i\delta_0}, \tag{60}$$

or

$$\Psi_1 = e^{\theta_1 + i\chi_1}, \tag{61}$$

$$\Psi_2 = e^{\theta_2 + i\chi_2}, \tag{62}$$

where $\mu_1 = \eta + i\nu$, $\lambda_1 = \alpha + i\beta$ and δ_i are real constants.

$$\theta_1 = \beta x - \nu y - [8\alpha\beta\eta + 4\nu(\alpha^2 - \beta^2) - \frac{\beta}{(\alpha+\mu)^2+\beta^2}]t + \delta_1, \quad (63)$$

$$\chi_1 = -\alpha x + \eta y + [4\eta(\alpha^2 - \beta^2) - 8\alpha\beta\nu + \frac{\alpha+\mu}{(\alpha+\mu)^2+\beta^2}]t + \delta_2, \quad (64)$$

and

$$\theta_2 = -\theta_1, \quad \chi_2 = -\chi_1 + \delta_0 \quad (65)$$

Finally we have one-soliton solution of the (2+1)-dimensional modified KdV equation:

$$q^{[1]} = \frac{2\beta e^{i\chi_1 - i\chi_2}}{ch2\theta_1}, \quad (66)$$

$$v^{[1]} = 16\frac{\alpha\beta\nu}{ch^2 2\theta_1} + 16\frac{\beta^2\eta}{ch^2 2\theta_1}, \quad (67)$$

$$w^{[1]} = -\frac{8\beta\nu}{ch^2 2\theta_1}, \quad (68)$$

$$p^{[1]} = \frac{2m_{12}(\mu + m_{11})}{\square}, \quad (69)$$

$$\eta^{[1]} = \frac{|\mu + m_{11}|^2 - |m_{12}|^2}{\square}, \quad (70)$$

where

$$m_{11} = \alpha + i\beta th2\theta_1, \quad m_{12} = \frac{i\beta e^{i\chi_1 - i\chi_2}}{ch2\theta_1}, \quad \square = \mu^2 + 2\alpha\mu + \alpha^2 + \beta^2 = (\mu+\alpha)^2 + \beta^2 \quad (71)$$

Using the above presented n-fold DT, in the same way we can construct the n-soliton solution of the (2+1)-dimensional modified KdV equation.

Conclusion

In this paper, we have constructed the DT for the (2+1)-dimensional modified KdV equation. Using the derived DT, some exact solutions like, one-soliton solution is obtained. The determinant representations of the obtained solutions of the (2+1)-dimensional modified KdV equation are given. Using the above presented results, one can also construct the n-solitons, breathers and rogue wave solutions of the (2+1)-dimensional modified KdV equation. It is interesting to note that rogue wave solutions of nonlinear equations is currently one of the hottest topics in nonlinear physics and mathematics. The application of the obtained solutions in physics will be one interesting subject [29-36]. In particular, we hope that the presented solutions may find some usages in experiment or optical fibre communication. Also we note that we will study some important generalizations of the (2+1)-dimensional modified KdV equation in future.

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Кемельбекова Г.М., Есмаханова К.Р., Тапеева С.К. Түнғүшбаева Д.И. және Мырзақұлов Р.

Дарбу түрлендіруі және(2+1)-өлшемді модификацияланған Кортвег де Фриз теңдеуінің бірсолитонды шешімі

Дарбу түрлендіруі бейсызықты эволюциялық теңдеулердің дәл шешімдерін құру үшін кешенді тәсі модификацияланған Кортвег де Фриз теңдеуінің шешімін құру үшін пайдаланылады. Біз осы мақалада кері шашыра есебі арқылы интегралданатын (2+1)-өлшемді модификацияланған Кортвег де Фриз теңдеуінің Дарбу түрлендіру құрдық. Осы түрлендіруді қолданып, „тривиалды" шешімдерден бірсолитонды шешім алдық.

Түйін сөздер: Дарбу түрлендіруі, солитондық шешім, Лакс жұптары, модификацияланған Кортвег де Фриз теңдеу

Кемельбекова Г.М., Есмаханова К.Р., Тапеева С.К., Тунгушбаева Д.И. и Мырзакулов Р.

Дарбу преобразование и односолитонное решение (2+1)-мерного модифицированного уравнени Кортвега де Фриза

Преобразование Дарбу, комплексный подход для построения точных решений нелинейного эволюционного уравнени применяется для построения решения модифицированного уравнения Кортевега - де Фриза (КдФ). В этой стать мы построили преобразование Дарбу (2 + 1) -мерного модифицированного уравнения КдФ, которое интегрирует по обратной задаче рассеяния. Используя это преобразование, односолитонное решение получено из «тривиальных решений.

Ключевые слова: Дарбу преобразование, солитонное решение, Лакс представление, модифицированное уравнен Кортвега де Фриза.

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Кемельбекова Г.М., Есмаханова К.Р., Тапеева С.К. Түнғүшбаева Д.И. және Мырзақұлов Р.
Дарбу түрлендіруі және(2+1)-өлшемді модификацияланған Кортвег де Фриз теңдеуінің бірсолитонд шешімі

Дарбу түрлендіруі бейсызықты эволюциялық теңдеулердің дәл шешімдерін құру үшін кешенді тәсі модификацияланған Кортвег де Фриз теңдеуінің шешімін құру үшін пайдаланылады. Біз осы мақалада кері шашыра есебі арқылы интегралданатын (2+1)-өлшемді модификацияланған Кортвег де Фриз теңдеуінің Дарбу түрлендіру құрдық. Осы түрлендіруді қолданып, „тривиалды“ шешімдерден бірсолитонды шешім алдық.

Түйін сөздер: Дарбу түрлендіруі, солитондық шешім, Лакс жұптары, модификацияланған Кортвег де Фриз теңдеу

Кемельбекова Г.М., Есмаханова К.Р., Тапеева С.К., Тунгушбаева Д.И. и Мырзакулов Р.
Дарбу преобразование и односолитонное решение (2+1)-мерного модифицированного уравнени Кортвега де Фриза

Преобразование Дарбу, комплексный подход для построения точных решений нелинейного эволюционного уравнени применяется для построения решения модифицированного уравнения Кортевега - де Фриза (КдФ). В этой стать мы построили преобразование Дарбу (2 + 1) -мерного модифицированного уравнения КдФ, которое интегрирует по обратной задаче рассеяния. Используя это преобразование, односолитонное решение получено из «тривиальны решений.

Ключевые слова: Дарбу преобразование, солитонное решение, Лакс представление, модифицированное уравнен Кортвега де Фриза.

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To satisfy the restriction of M and $B^{[1]}_{-1}$ as mentioned above, we first note that if $\delta = 1$ then

$$\Psi^{+} = \Psi^{-1}, \quad A_0^{+} = -A_0, \tag{45}$$

$$\lambda_2 = \lambda_1^*, \quad H = \begin{pmatrix} \psi_1(\lambda_1; t, x, y) & -\psi_2^*(\lambda_1; t, x, y) \\ \psi_2(\lambda_1; t, x, y) & \psi_1^*(\lambda_1; t, x, y) \end{pmatrix}, \tag{46}$$

$$H^{-1} = \frac{1}{\Delta} \begin{pmatrix} \psi_1^*(\lambda_1; t, x, y) & \psi_2^*(\lambda_1; t, x, y) \\ -\psi_2(\lambda_1; t, x, y) & \psi_1(\lambda_1; t, x, y) \end{pmatrix}. \tag{47}$$

So for the matrix M we have

$$M = \frac{1}{\Delta} \begin{pmatrix} \lambda_1|\psi_1|^2 + \lambda_2|\psi_2|^2 & (\lambda_1 - \lambda_2)\psi_1\psi_2^* \\ (\lambda_1 - \lambda_2)\psi_1^*\psi_2 & \lambda_1|\psi_2|^2 + \lambda_2|\psi_1|^2) \end{pmatrix}, \tag{48}$$

where

$$\Delta = |\psi_1|^2 + |\psi_2|^2. \tag{49}$$

Here we mention that $m_{22} = m_{11}^*$ and $m_{21} = -m_{12}^*$ if $\lambda_2 = \lambda_1^*$. So far, we found construction of DT of the (2+1)-dimensional modified KdV equation:

$$q^{[1]} = q - 2im_{12}, \tag{50}$$

$$v^{[1]} = v + 4(m_{12}q_y^* + m_{12}^*q_y + 2im_{11}m_{11y} - 2im_{12}^*m_{12y}), \tag{51}$$

$$w^{[1]} = w - 4im_{11y} = w + 4im_{11y}^*, \tag{52}$$

$$\eta^{[1]} = \frac{(|\mu + m_{11}|^2 - |m_{12}|^2)\eta - pm_{12}^*(\mu + m_{11}) - p^*m_{12}(\mu + m_{11}^*)}{\square}, \tag{53}$$

$$p^{[1]} = \frac{p(\mu + m_{11})^2 - p^*m_{12}^2 + 2\eta m_{12}(\mu + m_{11})}{\square}. \tag{54}$$

One-soliton solutions

As far as we got Darboux transformation, our next aim is to construct the one-soliton solution of KdV equation. In order to obtain soliton solution, the seed solution we take $q = 0$, $v = 0$, $p = 0$, $w = 0$, $\eta = 1$. Let $\lambda_1 = a + bi$. Hence the corresponding associated linear system takes the form

$$\Psi_{1x} = -i\lambda\Psi_1, \tag{55}$$

$$\Psi_{2x} = i\lambda\Psi_2, \tag{56}$$

$$\Psi_{1t} = 4\lambda^2\Psi_{1y} + \frac{i}{\lambda + \mu}\Psi_1, \tag{57}$$

$$\Psi_{2t} = 4\lambda^2\Psi_{2y} - \frac{i}{\lambda + \mu}\Psi_2. \tag{58}$$

From the above system, we take take eigenfunctions in the form

$$\Psi_1 = e^{-i\lambda_1 x + i\mu_1 y + i(4\lambda_1^2\mu_1 + \frac{1}{\lambda_1 + \mu})t + \delta_1 + i\delta_2}, \tag{59}$$

$$\Psi_2 = e^{i\lambda_1 x - i\mu_1 y - i(4\lambda_1^2\mu_1 + \frac{1}{\lambda_1 + \mu})t - \delta_1 - i\delta_2 + i\delta_0}, \tag{60}$$

or

$$\Psi_1 = e^{\theta_1 + i\chi_1}, \tag{61}$$

$$\Psi_2 = e^{\theta_2 + i\chi_2}, \tag{62}$$