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# Elastic scattering of low energy electrons in partially ionized dense semiclassical plasma

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Elastic scattering of electrons by hydrogen atoms in a dense semiclassical hydrogen plasma for low impact energies has been studied. Differential scattering cross sections were calculated within the effective model of electron-atom interaction taking into account the effect of screening as well as the quantum mechanical effect of diffraction. The calculations were carried out on the basis of the phase-function method. The influence of the diffraction effect on the Ramsauer–Townsend effect was studied on the basis of a comparison with results made within the effective polarization model of the Buckingham type. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4928877]

## I. INTRODUCTION

Interactions of charged particles with atoms and molecules are of great importance in many fields of physics. When electrons collide with atomic or molecular targets, a large variety of reactions can take place. In this work, we considered only low-energy impact elastic scattering of electrons by the hydrogen atoms in a dense semiclassical hydrogen plasma.

Most experiments on elastic scattering of electrons by atoms have been made in neutral gases.[1–3] There are also many theoretical investigations where the interaction of a flux of charged particles with atomic or molecular vapor is considered. The interaction potential of a charged particle with an atom (molecule) often consists of different parts: static, polarization, and exchange components.[4] The static part is related to the inner structure of the atom (molecule).[4,5] The polarization component appears when the electron is scattered by a particle with a dipole moment.[6,7] This dipole moment of the target atom can appear as a result of its polarization under the action of the slow charged projectile. The polarization potential, frequently used in many works, is the well-known Buckingham type potential[8]

$$\Phi_{ea}(r) = -\frac{e^2\alpha}{2\left(r^2+r_0^2\right)^2}, \tag{1}$$

where $\alpha$ is the dipole polarizability of the atom or molecule, $\alpha = 4.5\, a_B^3$ for the hydrogen atom, and $r_0 = (\alpha a_B/2)^{1/4}$ is the cutoff radius of the hydrogen atom, which was initially introduced to prevent the divergence at the starting point $r = 0$, where $a_B$ is the Bohr radius.

However, it is well known that the collective interactions between charged particles in the plasma lead to screening of the electric field around particles. It means that in plasma it is necessary to take into account the screening effect in pair interaction potentials of plasma particles. The screening version of the Buckingham potential, describing the interaction between the electron and the atom in plasma, is written as follows:[9]

$$\Phi_{ea}(r) = -\frac{e^2\alpha}{2\left(r^2+r_0^2\right)^2}\exp\left(-\frac{2r}{r_D}\right)\left(1+\frac{r}{r_D}\right)^2. \tag{2}$$

Here, $r_D = (k_B T/(8\pi e^2 n_e))^{1/2}$ is the Debye length, $n_e$ is the numerical density of electrons; $T$ is the plasma temperature; and $k_B$ is the Boltzmann constant. Recently, many studies of the collisional processes in plasma have been conducted taking into account the screening effect. It was shown that the scattering cross section significantly depends on such plasma parameters as plasma density and temperature.

It is well known that the incident and target electrons can exchange their places during collisions. To take into account this process, the exchange potentials are used.[10,11] These potentials quickly decrease with an increase in the distance between particles.

In addition, it must be noted that in the interaction of particles, their wave nature plays an important role. Therefore, the well-known Ramsauer-Townsend effect observed in the experiments is a convincing demonstration of the wave nature of particles. One of the first experimental investigations of the scattering cross section of electrons on $Ar, Kr, Xe$ atoms was conducted by Ramsauer and his group.[12] This phenomenon is considered to be one of the fundamental experimental proofs of the electron wave properties, and it was observed in the experiments on scattering of electrons not only by atoms of noble gases but also by atoms of other elements as well as by molecules.[1–7]

Theoretical calculations using equations of quantum mechanics to obtain scattering cross sections of electrons by atoms at some plasma parameters also demonstrated the emergence of the Ramsauer minimums.[13] Interaction potentials, used in such estimations, often do not take into account wave properties of particles. In this regard, it is interesting to carry out a theoretical investigation of the Ramsauer-Townsend effect based on the interaction potential directly taking into account the quantum-mechanical effect of diffraction. It is known that in non-ideal semiclassical plasma, the quantum mechanical effects, such as the diffraction and

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symmetry effects, take place. To take into account these quantum mechanical effects, a special method was developed; it is based on the comparison of a classical Boltzmann's factor and the quantum-mechanical Slater sum. This approach was first described in Ref. 14. The Deutsch interaction potential for charged particles,[15,16] taking into account the diffraction effect, has the following form:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{r}\left(1 - e^{-\frac{r}{\lambda_{\alpha\beta}}}\right). \quad (3)$$

Here, $\lambda_{\alpha\beta} = \hbar/\sqrt{2\pi m_{\alpha\beta} k_B T}$ is the de Broglie thermal wavelength; $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass of $\alpha$ and $\beta$ particles. Potential (3) has finite values at distances close to zero, and at large distances it tends to the Coulomb potential.

In Refs. 17–19, the effective potential for the interaction between charged particles, considering both screening and diffraction effects, was presented and successfully used

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{\sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2}}\left(\frac{e^{-B_{\alpha\beta} r}}{r} - \frac{e^{-A_{\alpha\beta} r}}{r}\right). \quad (4)$$

Here, $A_{\alpha\beta}^2 = \frac{1}{2\lambda_{\alpha\beta}^2}\left(1 + \sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2}\right)$; $B_{\alpha\beta}^2 = \frac{1}{2\lambda_{\alpha\beta}^2}\left(1 - \sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2}\right)$.

Recently, many papers, where dynamic models of the plasma particle interaction were considered, have been published (see Refs. 22–24). If the velocities of particles exceed the thermal velocity, they do not properly polarize the surrounding plasma and, as a result, weaken the screening. The screening, depending on the velocity of colliding particles, was called the dynamic screening, and now it is often used to study the non-ideal plasma properties. Dynamical potentials can be obtained by replacing the usual Debye length in the interaction potential by the screening radius, depending on the relative velocity of colliding particles. In Ref. 24, the elastic differential cross sections of charged particles in a dense semiclassical plasma were studied on the basis of the interaction potential taking into account the effects of diffraction and dynamic screening.

In this work, we used the effective potential for the electron-atom interaction presented in works[20,21,25] with static screening because we considered the low impact energy of electrons. It also takes into account the diffraction effect

$$\Phi_{ea}(r) = -\frac{e^2\alpha}{2r^4(1 - 4\lambda_{ea}^2/r_D^2)}\left(e^{-Br}(1 + Br) - e^{-Ar}(1 + Ar)\right)^2, \quad (5)$$

where

$$A^2 = \frac{1}{2\lambda_{ea}^2}\left(1 + \sqrt{1 - 4\lambda_{ea}^2/r_D^2}\right),$$
$$B^2 = \frac{1}{2\lambda_{ea}^2}\left(1 - \sqrt{1 - 4\lambda_{ea}^2/r_D^2}\right).$$

$\lambda_{ea} = \hbar/\sqrt{2\pi\mu_{ea} k_B T} \approx \lambda_e$ is the de Broglie thermal wavelength; $\mu_{ea} = m_e m_a/(m_e + m_a)$ is the reduced mass of the electron-atom pair. The potential (5) is screened and has finite values at small distances.

The main goal of this paper is to study the influence of the diffraction effect on elastic scattering of electrons by hydrogen atoms. For simplicity, we consider scattering of electrons only on the polarization potential. Such an approximation is acceptable at low energies of incident electrons, when the main contribution to the cross section is made by scattering on the polarization potential. Neither the static potential nor the exchange potential has been taken into account.

We have studied the elastic scattering of electron on the hydrogen atom on the basis of potential (5). A detailed study of the differential cross sections has been done in the energy range of 0.1–50 eV.

## II. METHOD AND PARAMETERS

To calculate the differential scattering cross sections of the electron on the hydrogen atom, the phase-function method was used. The basic equation of this method is called the Calogero equation and has the form[26]

$$\frac{d}{dr}\delta_l(k,r) = -\frac{1}{k}\frac{2m}{\hbar^2}\Phi_{\alpha\beta}(r) \times \left[\cos\delta_l(k,r) j_l(kr) - \sin\delta_l(k,r) n_l(kr)\right]^2, \quad (6)$$

$$\delta_l(k,0) = 0,$$

where $l$ is the orbital quantum number, $\delta_l(k,r)$ is the phase function, $\Phi_{\alpha\beta}(r)$ is the interaction potential of particles of $\alpha$ and $\beta$ species, $k$ is the wave number of the incident particle, and $j_l(kr)$ and $n_l(kr)$ are the Bessel functions. The phase shift is the asymptotical value of the phase function at large distances

$$\delta_l(k) = \lim_{r\to\infty}\delta_l(k,r). \quad (7)$$

Differential cross section is defined by the phase shifts as follows:[27]

$$\frac{d\sigma(k,\theta)}{d\Omega} = \left|\frac{1}{k}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)e^{i\delta_l(k)}\sin\delta_l(k)\right|^2. \quad (8)$$

Here, $P_l(\cos\theta)$ are the Legendre polynomials and $\theta$ is the scattering angle. In this work, the following dimensionless parameters were used for the hydrogen plasma:

$$\Gamma = \frac{e^2}{a\,k_B T} \quad (9)$$

is the coupling parameter, the average distance between particles is $a = (3/4\,\pi\,n)^{1/3}$; and

$$r_s = \frac{a}{a_B} \quad (10)$$

is the density parameter.

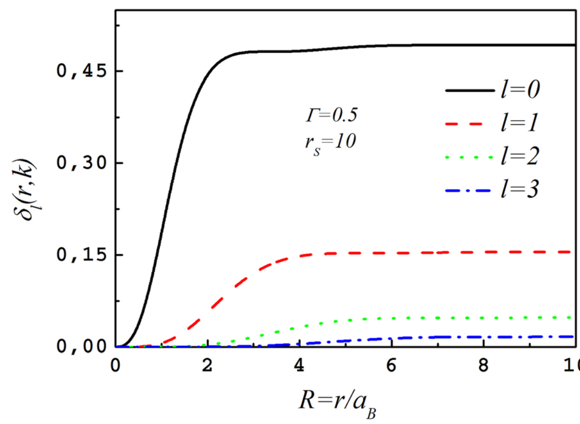


FIG. 1. Phase functions of electron-atom scattering in hydrogen plasma at $\Gamma = 0.5$, $r_s = 10$, and $k = 0.8a_B^{-1}$.

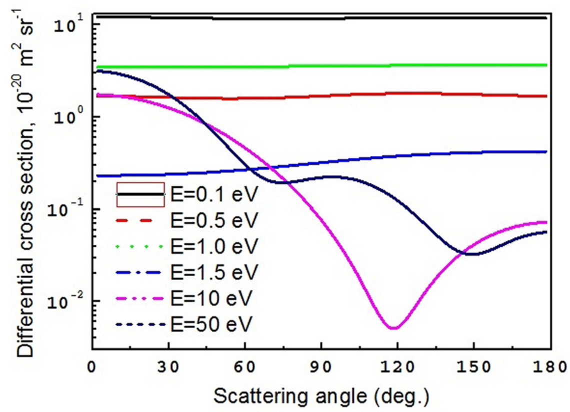


FIG. 3. Differential cross sections of the electron-atom scattering, obtained using potential (2), $\Gamma = 0.3$ and $r_s = 2$, as a function of the scattering angle.

## III. RESULTS

Equation (6) was solved numerically. Fig. 1 shows the dependence of phase functions of the electron-atom scattering on the distance obtained in the framework of potential (5). As we can see in Figure 1, the obtained phase functions demonstrate a proper asymptotic behavior, and at large distances they tend to some steady-state values, which are the phase shifts.

Figures 2 and 3 present the dependence of differential cross sections of the electron-atom scattering on the scattering angle obtained for potentials (5) and (2), respectively. The differential cross sections have minimums at low energies of the projectile electron. The energies, corresponding to the minimums, are smaller and the minimums are more pronounced in Fig. 2 than in Fig. 3, which may be explained by the fact that the interaction potential (5) takes into account the diffraction effect and the same wave nature is the basis of the Ramsauer-Townsend effect. Therefore, one can conclude that consideration of the diffraction effect in the interaction potential gives a more visible manifestation of the Ramsauer-Townsend effect in comparison with the model of the Buckingham potential (2) in the dense semiclassical hydrogen plasma.

To describe the influence of the surrounding plasma on the collision of two particles, the differential scattering cross sections, shown in Fig. 4, were obtained in the framework of the model (5) for different values of the coupling parameter. This figure indicates that an increase in the coupling parameter (an increase in the width and depth of the potential well around the atom) causes a decrease in the impact energy corresponding to the minimum. This is due to the fact that the de Broglie wavelength of the incident electron, at which the Ramsauer-Townsend effect appears, is proportional to the width of the potential well. Fig. 5 shows the differential scattering cross section obtained on the basis of the model (5) for different values of the density parameter. The width of the potential well also depends on the density because strong screening at high densities reduces the width of the well and, respectively, increases the impact energy corresponding to the minimum.

Fig. 6 shows the dependence of the differential cross sections of the electron scattering by the hydrogen atom, obtained using potential (5), on the energy of the projectile electron for different values of the scattering angle. It is seen that an increase in the scattering angle causes a decrease in the impact energy corresponding to the minimum. Fig. 7

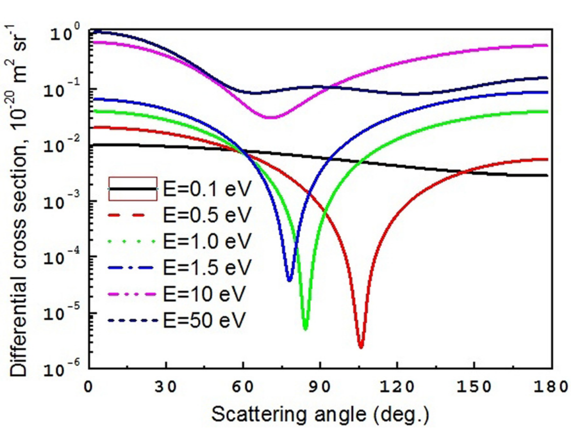


FIG. 2. Differential cross sections of the electron-atom scattering, obtained using potential (5) $\Gamma = 0.3$ and $r_s = 2$, as a function of the scattering angle.

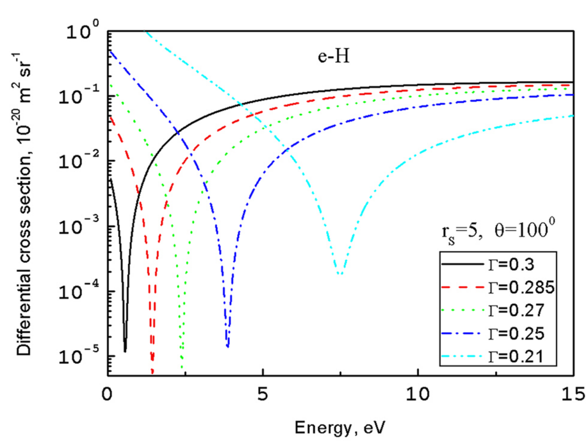


FIG. 4. Differential cross sections of the electron-atom scattering, obtained using potential (5), as a function of impact energy for different values of the coupling parameter.

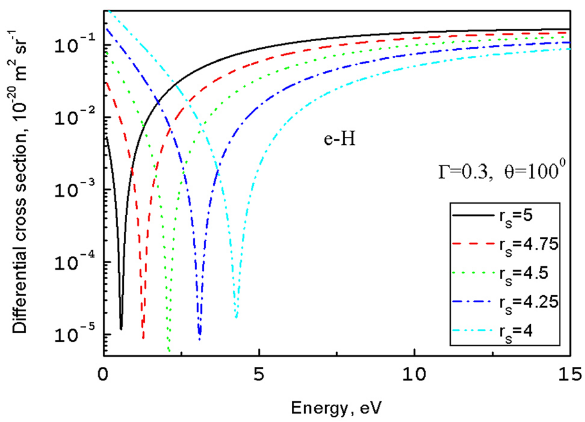


FIG. 5. Differential cross sections of the electron-atom scattering, obtained using potential (5), as a function of the impact energy for different values of the density parameter.

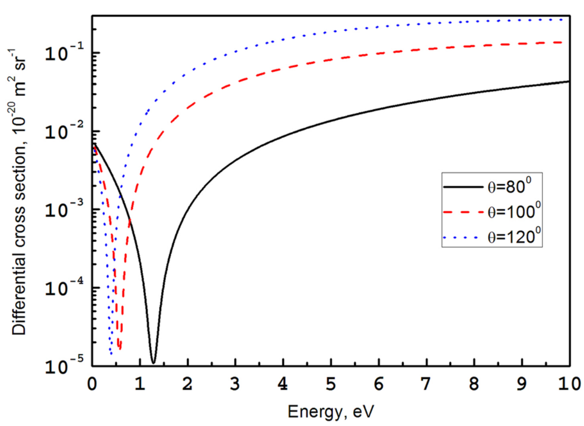


FIG. 6. Differential cross sections of the electron-atom scattering, obtained using potential (5), as a function of the energy of the projectile electron, $\Gamma = 0.3$ and $r_s = 5$.

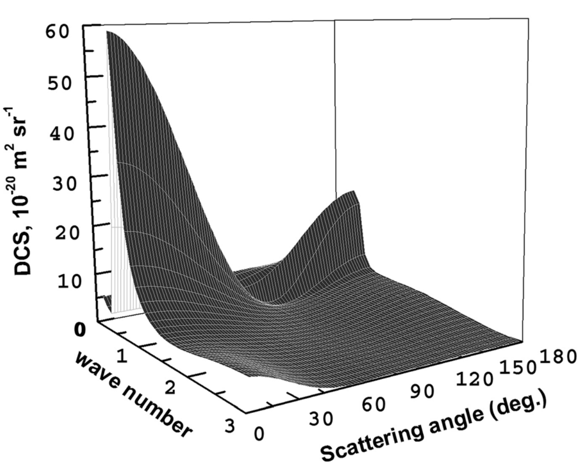


FIG. 7. Differential cross section of the electron-atom scattering, obtained using potential (5), as a function of energy of the projectile electron and the scattering angle, $\Gamma = 0.3$ and $r_s = 5$.

shows the differential scattering cross section obtained using potential (5) as a function of the energy of the projectile electron and the scattering angle. One can see the formation of a local minimum region on the surface.

## IV. SUMMARY

In this work, the differential scattering cross sections of electrons on the hydrogen atoms at low impact energies were calculated on the basis of the interaction polarization potential taking into account the screening effect and the quantum mechanical diffraction effect.

The results, demonstrating the Ramsauer minimums, were compared with the results obtained within the well-known polarization potential of the Buckingham type. On the basis of this comparison, one can conclude that taking into account the diffraction effect in the interaction potential leads to a decrease in the energy at which the Ramsauer-Townsend effect is observed. This conclusion can be made for the scattering processes of electrons by the atoms of other elements. This problem is now under our consideration.

Another important conclusion is that a decrease in the temperature or density of plasma causes a decrease in the impact energy of scattered electrons corresponding to the minimum.

Such research has important applications in astrophysics and plasma physics. The obtained results can be used for calculation of the dynamic properties of the semiclassical dense plasma. Knowledge of these properties plays a major role in the design of technical installations using dense nonideal plasma.

## ACKNOWLEDGMENTS

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