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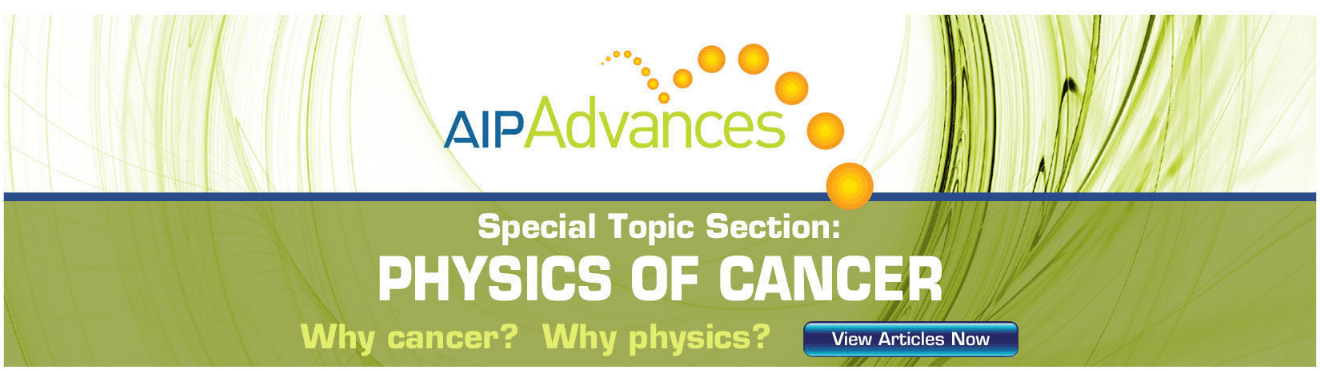

Dynamic interaction potential and the scattering cross sections of the semiclassical plasma particles

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The dynamic model of the charged particles interaction in non-ideal semiclassical plasma is presented. This model takes into account the quantum mechanical diffraction effect and the dynamic screening effect. On the basis of the dynamic interaction potential, the electron scattering cross sections are investigated. Comparison with the results obtained on the basis of other models and conclusions were made. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4799798]

I. INTRODUCTION

Development of the particle interaction models and the researches of the non-ideal dense plasmas properties on their basis are of a great fundamental and practical interest (see, for example, Refs. 1–8). It is important for development of the technologies of the many practical applications connected with non-ideal plasma, for example, thermonuclear fusion by the laser compression, MGD-generator, and others. One of the main problems of plasma using is a problem of its confinement and controlling; therefore, knowledge of transport coefficients, such as diffusion, thermal conductivity, electro conductivity, and viscosity coefficients have paramount significance for designing of technological installations.

Historically, the first pseudo-potential model describing the screening effect in non-ideal plasma was the Debye-Hückel model

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{r}\exp\left(-\frac{r}{r_D}\right). \tag{1}$$

Here, r is the distance between particles, $r_D = (k_B T/(4\pi e^2 \sum_j n_j Z_j^2))^{1/2}$ is the Debye length, $Z_\alpha e$, $Z_\beta e$ are the electrical charges of α and β sorts.

In Ref. 9, it was shown that the Debye-Hückel potential corresponds to approach of pair correlations. It is known that in non-ideal semiclassical plasma besides correlations the quantum mechanical effects of diffraction and symmetry take place. To take into account quantum mechanical effects a special method was developed, it consists of the comparison of a classical Boltzmann's factor and the quantum mechanical Slater sum. This approach was first described in Ref. 10. The Deutsch pseudo-potential,[7,8] which considers the diffraction effect, has the following form:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{r}\left(1 - e^{-\frac{r}{\lambda_{\alpha\beta}}}\right). \tag{2}$$

Here, $\lambda_{\alpha\beta} = \hbar/\sqrt{2\pi m_{\alpha\beta} k_B T}$ is the de Broglie thermal wavelength; $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$—is the reduced mass of α and β interacted particles.

In works of Refs. 1 and 2, the effective potential for electron–charge interaction considering both effects of screening and diffraction was presented:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{\sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2}}\left(\frac{e^{-B_{\alpha\beta} r}}{r} - \frac{e^{-A_{\alpha\beta} r}}{r}\right). \tag{3}$$

Here $A_{\alpha\beta}^2 = \frac{1}{2\lambda_{\alpha\beta}^2}(1 + \sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2})$; $B_{\alpha\beta}^2 = \frac{1}{2\lambda_{\alpha\beta}^2}(1 - \sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2})$.

As one can see this potential is valid only for $r_D^2 > 4\lambda_{\alpha\beta}^2$, potential (3) is screened and also has finite values at distances close to zero. The ion-charge and electron-atom interaction potentials for particles of the dense complex plasmas were obtained in works.[3,5]

Recently, a lot of works on creation of dynamic models of the plasma particles interaction and plasma properties researches in frameworks of these models were performed (see Refs. 11–13). These models consider relative velocities of colliding particles. It should be noted that collision cross sections directly depend on the relative velocity. It is clear that the application of the dynamic potentials of the particles interaction at studying of their collisions is more correct. Section II presents the dynamic interaction potential of the charged particles of the semiclassical plasma, which takes into account diffraction and dynamical screening effects.

Electron elastic scattering remains interesting problem due to its influence on the kinetic properties and wide application in the diagnostics of the properties of different objects.[14–17] In Section III, the electron scattering cross sections obtained on the basis of the presented dynamic potential are investigated and discussed.

II. THE PARAMETERS AND THE DYNAMIC INTERACTION POTENTIAL OF THE SEMICLASSICAL PLASMA PARTICLES

For the description of the non-ideal plasma properties, it is convenient to use the dimensionless parameters characterizing plasma state at the certain densities and temperatures. One of the parameters is the coupling parameter

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$$\Gamma_{\alpha\beta} = \frac{Z_\alpha Z_\beta e^2}{a\, k_B T}. \tag{4}$$

Here, the average distance between particles is

$$a = \left(\frac{3}{4\pi n}\right)^{1/3}. \tag{5}$$

Here, $n = n_e + n_i$ is the numerical density of electrons and ions; T is the plasma temperature; k_B is the Boltzmann's constant. The density parameter is determined as

$$r_s = \frac{a}{a_B}. \tag{6}$$

Here, $a_B = \frac{\hbar^2}{m_e e^2}$ is the Bohr radius. The density parameter diminishes with increase in plasma density.

The degeneration parameter for electron component is

$$\Theta = \frac{k_B T}{E_F} = 2\left(\frac{4}{9\pi}\right)^{\frac{2}{3}} Z^{\frac{5}{3}} \frac{r_s}{\Gamma_{ee}}. \tag{7}$$

Here, E_F is the Fermi's energy.

It is necessary to note that traditionally screening of the charge electric field in plasma is represented by the static screening of the Debye–Hückel. This approach is valid if the velocities of colliding particles are close to the thermal velocity. If velocities of particles exceed the thermal velocity such fast particles do not properly polarize surrounding plasma and as a result, weaken screening. Screening, depending on the velocity of colliding particles was called the dynamic screening, and now it is often used to study the non-ideal plasma properties. In Ref. 11, the method of accounting of dynamic screening was presented, it is reduced to the replacement of the static Debye length by an effective length connected with dynamic screening

$$r_0 = r_D\left(1 + \frac{v^2}{v_{Th}^2}\right)^{1/2}. \tag{8}$$

Here, v is the relative velocity of colliding particles and v_{Th} is the thermal velocity. Then the pseudo-potential (3) for electron-electron interaction, which takes into account dynamic screening, in a dimensionless form is

$$\Phi_{ee}(R)/k_B T = \frac{\Gamma_{ee}}{\sqrt{1 - 24\Gamma_{ee}^2/(\pi r_s(1+\delta^2))}}\left(\frac{e^{-B_{ee}R}}{R} - \frac{e^{-A_{ee}R}}{R}\right), \tag{9}$$

where

$$A_{ee}^2 = \frac{\pi r_s}{4\Gamma}\left(1 + \sqrt{1 - 24\Gamma_{ee}^2/(\pi r_s(1+\delta^2))}\right),$$

$$B_{ee}^2 = \frac{\pi r_s}{4\Gamma}\left(1 - \sqrt{1 - 24\Gamma_{ee}^2/(\pi r_s(1+\delta^2))}\right),$$

$\delta = v/v_{Th}$ is the parameter of the relative velocity, $R = r/a$ is the distance between particles in terms of the average distance.

In Figures 1 and 2, pair interaction potentials between electrons are presented: the Coulomb, Deutsch, and Debye-Hückel potentials, the potential (3), considering the static screening, and

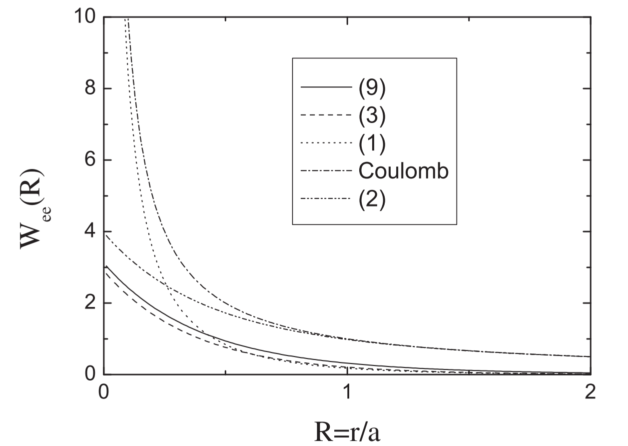

FIG. 1. Interaction potentials $W_{ee}(R) = \Phi_{ee}(R)/k_B T$ of electron-electron pair for $\Gamma_{ee} = 1$, $r_s = 10$, and $\delta = 1$.

the potential (9), considering the dynamic screening. Apparently from these figures, the potential (9) at small velocity tends to the potential (3), and at large velocities tends to the Deutsch potential, which does not consider the screening.

III. DIFFERENTIAL SCATTERING CROSS SECTIONS OF THE ELECTRONS

Data on the differential scattering cross sections are a basis for calculation of the transport coefficients of the dense plasma. For high-temperature semiclassical plasma the differential scattering cross sections of the plasma particles can be qualitatively investigated on the basis of the Born formula (see Ref. 18)

$$\sigma(\theta, k) = \frac{4\pi m^2}{h^4}\left|\int \Phi_{ee}(r) e^{i(\vec{k}-\vec{k}_0)\vec{r}} d\vec{r}\right|^2 2\pi \sin\theta d\theta, \tag{10}$$

where $\Phi_{ee}(r)$ is the pair interaction potential of the particles, $\vec{k}_0$ and $\vec{k}$ are the wave vectors of moving particle (projectile) before collision and after (at elastic scattering $k = k_0$), θ is a scattering angle, which for elastic scattering is connected with wave vectors by the following expression $|\vec{k} - \vec{k}_0| = q = 2k\, sin\frac{\theta}{2}$.

In Refs.19 and 20, particle collisions and transport properties of semiclassical dense plasma were studied on the basis of the model (3), taking into account both a static

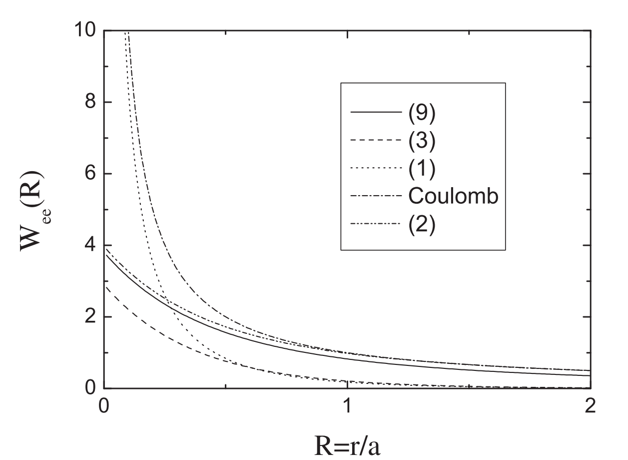

FIG. 2. Interaction potentials $W_{ee}(R) = \Phi_{ee}(R)/k_B T$ of electron-electron pair for $\Gamma_{ee} = 1$, $r_s = 10$, and $\delta = 10$.

screening and diffraction effect. The corresponding formula for differential scattering cross section was obtained. It can be rewritten in a dimensionless form as follows:

$$\begin{aligned}
\sigma(\theta,\kappa)/a^2 &= \frac{4r_s^2}{\left(1-\dfrac{24\Gamma_{ee}^2}{\pi r_s}\right)}\left(\frac{1}{A_{ee}^*+C}-\frac{1}{B_{ee}^*+C}\right)^2,\\
A_{ee}^* &= \frac{\pi r_s}{4\Gamma_{ee}}\left(1-\sqrt{1-\frac{24\Gamma_{ee}^2}{\pi r_s}}\right),\\
B_{ee}^* &= \frac{\pi r_s}{4\Gamma_{ee}}\left(1+\sqrt{1-\frac{24\Gamma_{ee}^2}{\pi r_s}}\right),\\
C &= 4(k^*)^2\sin^2\frac{\theta}{2};\quad k^*=ka.
\end{aligned}\tag{11}$$

In order to calculate the differential scattering cross section of electrons on charged particles in terms of the dynamic model of interaction (9) on the basis of the Born method the following expression was obtained:

$$\begin{aligned}
\sigma(\theta,\kappa)/a^2 &= \frac{4r_s^2}{\left(1-\dfrac{24\Gamma_{ee}^2}{\pi(r_s+(k^*)^2\Gamma_{ee})}\right)}\left(\frac{1}{A_{ee}^*+C}-\frac{1}{B_{ee}^*+C}\right)^2,\\
A_{ee}^* &= \frac{\pi r_s}{4\Gamma_{ee}}\left(1-\sqrt{1-\frac{24\Gamma_{ee}^2}{\pi(r_s+(k^*)^2\Gamma_{ee})}}\right),\\
B_{ee}^* &= \frac{\pi r_s}{4\Gamma_{ee}}\left(1+\sqrt{1-\frac{24\Gamma_{ee}^2}{\pi(r_s+(k^*)^2\Gamma_{ee})}}\right),\\
C &= 4(k^*)^2\sin^2\frac{\theta}{2},
\end{aligned}\tag{12}$$

where the ratio of the square of the projectile velocity to the square of the thermal velocity is expressed through the coupling and density parameters and a dimensionless wave vector, which defines the projectile momentum

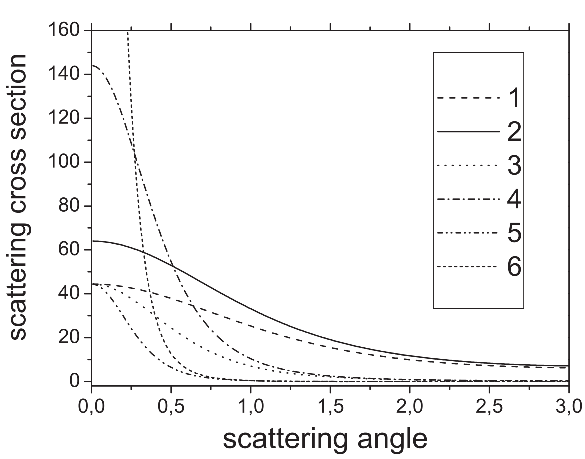

FIG. 3. Electron scattering cross sections reduced by a^2, $\Gamma_{ee}=1$, and $r_s=10$. 1—on the basis of the potential (3), k = 1, 2—on the basis of the potential (9), k = 1, 3—on the basis of the potential (3), k = 2, 4—on the basis of the potential (9), k = 2, 5—on the basis of the potential (3), k = 4, and 6—on the basis of the potential (9), k = 4.

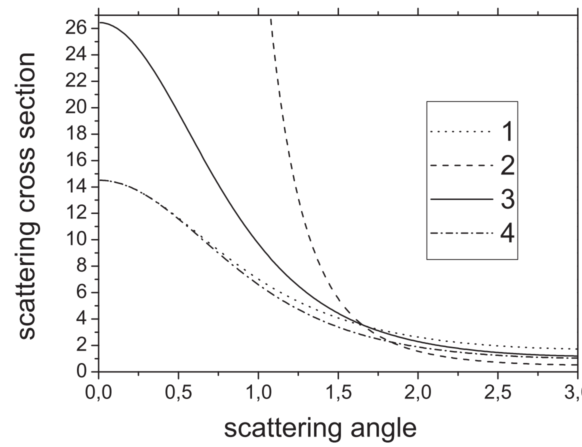

FIG. 4. Electron scattering cross sections $\Gamma_{ee}=0.7$, $r_s=4$, and $k=1$. 1—on the basis of the potential (1), 2—on the basis of the Deutsch potential (2), 3—on the basis of the potential (9), and 4—on the basis of the potential (3).

$$\left(\frac{\upsilon^2}{\upsilon_{Th}^2}\right)=\frac{\Gamma_{ee}}{r_s}(k^*)^2;\quad k^*=ka. \tag{13}$$

Figure 3 presents the electron scattering cross sections calculated by formulas (11) and (12) for different values of the wave vector. The Figure 3 shows that the cross sections obtained on the basis of the potential (3) have the same finite value at the scattering angle equal to zero because screening in model (3) does not depend on the velocity (momentum) of the projectile. Meanwhile, the scattering cross sections obtained within the potential (9) at the small scattering angle have finite values depending on the projectile momentum. The more the wave vector, the faster the cross section decreases with growth of the scattering angle, and finite value at $\theta\to 0$ becomes larger, which is caused by weakening of screening. From comparison of Figures 4 and 5 it is seen that with increase in the velocity of projectile the results within the dynamic potential tend to the data obtained on the basis of the Deutsch potential. In Figures 6 and 7, the comparison of the electron scattering cross sections calculated at the different values of the coupling and density parameters are shown. All curves behave absolutely adequately.

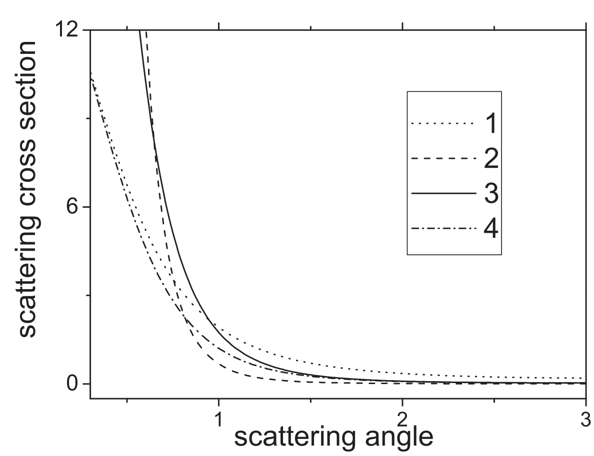

FIG. 5. Electron scattering cross sections $\Gamma_{ee}=0.7$, $r_s=4$, and $k=2$. 1—on the basis of the potential (1), 2—on the basis of the Deutsch potential (2), 3—on the basis of the potential (9), and 4—on the basis of the potential (3).

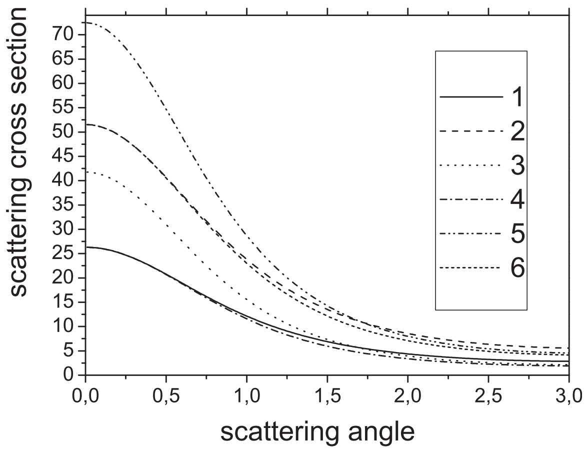

FIG. 6. Electron scattering cross sections for different values of the density parameter, $\Gamma_{ee} = 0.65$ and $k = 1$. 1—on the basis of the potential (1), $r_s = 5$, 2—on the basis of the potential (1), $r_s = 7$, 3—on the basis of the potential (9), $r_s = 5$, 4—on the basis of the potential (3), $r_s = 5$, 5—on the basis of the potential (9), $r_s = 7$, and 6—on the basis of the potential (3), $r_s = 7$.

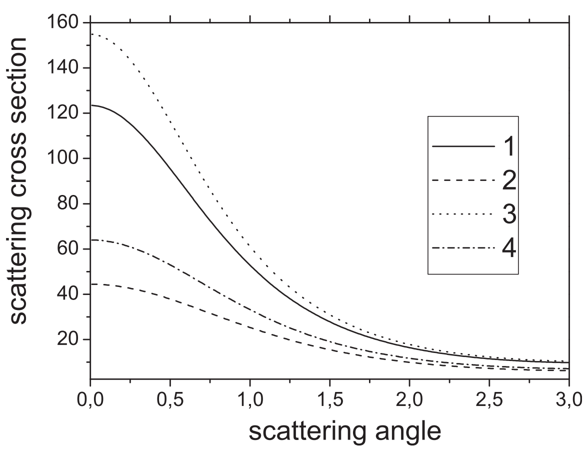

FIG. 7. Electron scattering cross sections for different values of the coupling parameter, $r_s = 10$ and $k = 1$. 1—on the basis of the potential (3), $\Gamma_{ee} = 0.6$, 2—on the basis of the potential (3), $\Gamma_{ee} = 1$, 3—on the basis of the potential (9), $\Gamma_{ee} = 0.6$, and 4—on the basis of the potential (9), $\Gamma_{ee} = 1$.

IV. CONCLUSION

The dynamic model of interaction of the non-ideal semi-classical plasma particles interaction was constructed. In this model, the dynamic screening which is weaker than the static screening was considered. The length of dynamic screening depends on the velocity of the projectile. This model also takes into account the diffraction effect manifested at small distances between particles. The dynamic potential at small relative velocities tends to the effective potential with the static screening (3), and at large velocities tends to the Deutsch potential, which does not consider screening.

In terms of the dynamic model, the important characteristic of collision processes, such as differential scattering cross sections, has been studied. The analysis of obtained results showed that differential cross sections on the basis of dynamic potential at small velocities of projectile on small scattering angle have finite values depending on velocity and at large angles tend to data based on the Deutsch potential. With increase in the velocity, results obtained on the basis of the dynamic potential are close to the data obtained on the basis of the Deutsch potential, not only on large scattering angles but also on small ones.

ACKNOWLEDGMENTS

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