BOOK OF ABSTRACTS

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Thus, in this talk we consider the problem of universality in a more general context than that of equivalence relations. First, we prove that, contrary to the case of equivalence relations, for each level of the arithmetical hierarchy there is a universal binary relation. Then we show how to make use of this latter construction in order to obtain a similar result also for preorders (i.e., reflexive and transitive binary relations) and graphs (i.e. symmetric binary relations).

[1] Uri Andrews, Steffen Lempp, Joseph S. Miller, Keng Meng Ng, Luca San Mauro, Andrea Sorbi, *Universal computably enumerable equivalence relations*, ***The Journal of Symbolic Logic***, vol. 79 (2014), no. 1, pp. 60–88.

[2] Egor Ianovski, Russell Miller, Keng Meng Ng, André Nies, *Complexity of equivalence relations and preorders from computability theory*, ***The Journal of Symbolic Logic***, vol. 79 (2015), no. 3, pp. 859–881.

▸ SERGEY OSPICHEV, *Computable numberings of partial computable functionals*.
Sobolev Institute of Mathematics and Novosibirsk State University, Novosibirsk, Russia.
E-mail: `ospichev@gmail.com`.

Study the cardinality and the structure of Rogers semilattices of families of various objects is one of the main questions in numbering theory. Here we concentrate our interest on partial computable functionals of finite types.

Let's define *functional type*. Let T will be the set of all types.

1. $0 \in T$;
2. if σ,τ are types, then $(\sigma \times \tau)$ and $(\sigma|\tau)$ are also types;
3. T - minimal set, satisfying 1 and 2.

Now we define *partial computable functionals*. Let C_σ be family of all partial computable functionals of type σ. Let $C_0 \rightleftharpoons \mathcal{C}$ be the family of all partial computable functions. If C_σ and C_τ are already defined, then $C_{(\sigma\times\tau)} \rightleftharpoons C_\sigma \times C_\tau$ and $C_{(\sigma|\tau)} \rightleftharpoons \mathfrak{Mor}(C_\sigma, C_\tau)$.

In work are proven

Theorem. *For any $\sigma \in T$ there is friedberg numbering of family C_σ.*

Theorem. *For any $\sigma \in T$ there is positive undecidable numbering of family C_σ.*

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▸ MANAT MUSTAFA, *Reductions between Types of Numberings*.
Department of Mathematics, Nazarbayev University, Qabanbay Batyr Ave 53., Astana, 010000, Kazakhstan.
E-mail: `manat.mustafa@nu.edu.kz`.

The theory of numberings is one of the fundamental topics in computability theory and mathematical logic. It is basically due to Gödel's idea to code countable families of objects by numbers, so that objects of the family can be effectively identified with numbers, or indices, and studied from their indices. While numberings are a powerful tool to use the set of natural numbers in order to study families of constructive objects, they are an interesting object of study in themselves: Here, an important device is that of reducibility between numberings, where a numbering is reducible to another numbering, if there is an effective way to go from indices of an object in the first numbering to indices of the same object in the second numbering. Thus the relative complexity of numberings of objects of a same family can be measured by this notion of reducibility, and gives rise to the so called Rogers upper semilattice of the family, whose elements are the degrees of numberings; H. Rogers[1] initiated the study of the semilattice of numberings under many-one reduction and Ershov [4, 5, 6] transferred it in particular to the study of the k-r.e. and, more generally, α-r.e. sets. The overall goal of this talk is to show some reductions between various types of numberings:

- If a k-r.e. numbering can realise a certain type of Rogers semilattice, so can a $(k+1)$-r.e. numberings or, more general, every $(\alpha+k)$-r.e. numbering where α is a computable ordinal;
- Every type of Rogers semilattice realised by an r.e. numbering is also realised by an α-r.e. for every computable ordinal α which is not a power of ω and which is not 0 while if α is a power of ω then there is no α-r.e. numbering without minimal numberings in the Rogers semilattice (which stands in contrast to the r.e. case);

This is joint work with F. Stephan and Ian Herbert from National University of Singapore.

[1] H. Rogers, *Gödel numberings of partial computable functions.*, ***J. Symbolic Logic,***,1958, v. 23, no. 3, pp. 4957.

[2] S. Badaev and S. Goncharov., *The theory of numberings: open problems.*, ***Computability Theory and its Applications*** (P. A. Cholak, S. Lempp, M. Lerman, and R. A. Shore, editors), American Mathematical Society, Providence, vol. 257, 2000, pp. 23–38.

[3] S. Goncharov and A. Sorbi, *Generalized computable numerations and non-trivial Rogers semilattices*, ***Algebra and Logic***, vol. 36 (1997), no. 6, pp. 359–369

[4] Yuri L. Ershov., *A certain hierarchy of sets I* , ***Algebra i Logika*** , 7(1):47–74, 1968.

[5] Yuri L. Ershov., *A certain hierarchy of sets II* , ***Algebra i Logika*** , 7(4):15–47, 1968.

[6] Yuri L. Ershov., *A certain hierarchy of sets III* , ***Algebra i Logika*** ,9:34–51, 1970.

▸ ASSYLBEK ISSAKHOV, *A–computable numberings of the families of total functions*.
Department of Mechanics and Mathematics, Al-Farabi Kazakh National University, 71 Al-Farabi Ave., Almaty 050038, Kazakhstan.
E-mail: `asylissakhov@mail.ru`.

Following [1], we say that a numbering $\nu : \omega \mapsto \mathcal{F}$ of a family of A–computable functions is A–computable if the binary function $\nu(n)(x)$ is A–computable. In [2], it was posed several natural questions on numberings that are computable relative to an arbitrary oracle. We give answers for some of them below.

Theorem 1. *Let A be an arbitrary set and F be an infinite A–computable family of total functions. If F has at least two non-equivalent A–computable Friedberg numberings, then F has infinitely many pairwise non-equivalent A–computable Friedberg numberings.*

Theorem 2. *Let A be a hyperimmune set. If A–computable family F of total functions contains at least two functions, then F has no principal A–computable numbering.*

Remind, [3], that every nonzero degree comparable with $0'$ is hyperimmune.

Note that, for every A such that $\emptyset' \leq_T A$, it was shown, [4], that an infinite A–computable family F of total functions has, up to equivalence, infinitely many A–computable Friedberg numberings; and if F contains at least two functions, then F has no principal A–computable numbering.

[1] S. S. Goncharov and A. Sorbi, *Generalized computable numerations and non-trivial Rogers semilattices*, ***Algebra and Logic***, vol. 36 (1997), no. 6, pp. 359–369.

[2] S. A. Badaev and S. S. Goncharov, *Generalized computable universal numberings*, ***Algebra and Logic***, vol. 53 (2014), no. 5, pp. 355–364.

[3] W. Miller and D. A. Martin, *The degree of hyperimmune sets*, ***Z. Math. Logik Grundlag. Math.***, vol. 14 (1968), pp. 159–166.

[4] A. A. Issakhov, *Ideals without minimal elements in Rogers semilattices*, ***Algebra and Logic***, to appear.