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ON THE THEORY OF TURBULENT HEAT AND MASS TRANSFER WITH ALLOWANCE FOR INTERMITTENCE EFFECTS

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A new viewpoint of the mechanism of turbulent heat and mass transfer has been proposed; a mathematical apparatus of statistical theory and a method of construction of mathematical models under the conditions of intermittent dynamic and scalar fields of nonuniform turbulent flow have been developed. A model of a turbulent axisymmetric jet for evaluation testing of the method has been constructed and the conditional and total means of the basic statistical characteristics of the jet have been calculated. It has been shown that the reason for the "different mechanism" of turbulent transfer of momentum, heat, and substance is the intermittence of not coincident dynamic and scalar fields of turbulent flow. An equation for the conditional probability density function of the concentration of a passive impurity has been derived and its solution for the axisymmetric jet in a strongly intermittent region has been given. Good agreement between the performed calculations and experimental data confirms the adopted concept of the transfer mechanism.

Keywords: *turbulence, heat and mass transfer, statistical theory, mathematical modeling, probability density function.*

Introduction. Increasingly growing requirements on the accuracy of calculation of turbulent flows make it necessary to improve mathematical-modeling methods. This is also demonstrated by the results of regular testing of models of such flows, which are assessed now as being "not quite satisfactory" (see, e.g., [1]). The problem becomes even more pressing in the case of flows with the nonuniform field of a scalar substance: it is well known, e.g., that modeling of the turbulent transfer of heat and substance is accompanied by the greatest errors in calculations. As a consequence this has a negative effect on turbulent-combustion models, particularly when they are constructed on the basis of the hydrodynamic characteristics of the concentration field of a passive impurity with the methods of the probability density function (PDF) [2, 3, and others]. Therefore, it seems quite topical to develop a theory of turbulent heat and mass transfer and a new, more efficient method of constructing mathematical models of turbulent flows with nonuniform scalar fields based on it.

Analysis of the Problem. It is generally agreed that the transfer of a scalar substance (heat and/or substance as a dynamically passive impurity) in nonuniform turbulent flows with in-plane shear occurs with a higher rate than the transfer of momentum [see, e.g., [4]). This conclusion is inferred from an analysis of the profiles for the total (unconditionally averaged) mean values of velocity and temperature (concentration of the passive impurity); the latter is always wider as far as its conditional boundaries are concerned. The reasons for the above difference have not been established yet, whereas the absence of a clear, physically substantiated mechanism of transfer of a scalar substance together with the development of an adequate statistical theory hinders the construction of efficient mathematical models of nonuniform turbulent flow.

Introduction of turbulent Prandtl $\mathrm{Pr_t}$ and Schmidt $\mathrm{Sc_t}$ numbers is assumed to be a traditional technique in modeling the turbulent transfer of momentum, energy (heat), and substance (concentration) with the hypotheses of closure of a gradient character. Thus, in the simplest case (selected here for clear representation) of nonisothermal and, on the average, two-dimensional flow described by the system of equations

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$$\langle u\rangle \frac{\partial\langle u\rangle}{\partial x}+\langle v\rangle \frac{\partial\langle u\rangle}{\partial y}=-\frac{1}{\rho}\frac{\partial\langle P\rangle}{\partial x}-\frac{\partial\langle u'v'\rangle}{\partial y}\,,\quad -\langle u'v'\rangle=\nu_{\mathrm{t}}\frac{\partial\langle u\rangle}{\partial y}\,, \tag{1}$$

$$\langle u\rangle \frac{\partial\langle T\rangle}{\partial x}+\langle v\rangle \frac{\partial\langle T\rangle}{\partial y}=-\frac{\partial\langle v'T'\rangle}{\partial y}\,,\quad -\langle v'T'\rangle=a_{\mathrm{t}}\frac{\partial\langle T\rangle}{\partial y}\,, \tag{2}$$

the introduction of the turbulent number $\mathrm{Pr_t} = \nu_t/a_t$ enables us to substantially simplify the problem of modelling. In this approach, selection of $\mathrm{Pr_t} < 1$ (usually from the range 0.5–0.9) because of the "wider" profile of the mean temperature makes it possible to somewhat set up a correspondence between the results of calculation of the mean values $\langle T\rangle$ and experimental data and thus to allow for the "different mechanism" of turbulent transfer of momentum and heat. However, the absence of a physically substantiated concept reduces such a technique to forcing the calculation results to fit experimental data, whereas the introduction of "turbulent numbers" into the semiempirical theory of turbulent transfer turns out to be forced. Therefore, it comes as no surprise that even such fitting does not work well: errors in calculations of the "mean" and particularly "pulsation" (central mixed moments of different orders) characteristics of the transferred scalar substance become inadmissibly great, primarily in regions of strongly intermittent flow.

On the Necessity of Allowing for Intermittence Effects. We observe at present the stage of going from the classical method of construction of turbulent-flow models according to RANS, that is based on Reynolds equations of the (1) and (2) type, to a method ensuring construction of more detailed and accurate models due to the allowance for intermittence effects.[1] The constructed models make it possible to calculate both the total and conditional mean characteristics of each of the intermittent media of turbulent flow. However modeling of the turbulent transfer of a scalar substance involves certain difficulties. They make themselves evident mainly in the fact that the "boundaries" of two different fields — the nonuniform field of the substance and the dynamic field of the turbulent liquid — are not coincident. This fact is confirmed by numerous experimental data and is explained by the nonlocal character of pressure pulsations [2]. The "capture" of a uniform (in concentration and/or temperature) nonturbulent liquid from the "external" medium by large vortices and the "diastrophism" of the "tongues" of such a liquid that have thus been entrained into the turbulent flow contributes to the generation of turbulence under the action of the velocity shift and pressure pulsations. Since the scalar field of the tongues is uniform, the velocity shift and pressure pulsations inside these "tongues" generate only velocity pulsations without producing pulsations of the scalar substance itself. It is precisely for this reason that the coefficients of intermittence of the dynamic (by alternation of the regions with turbulent and nonturbulent liquids) and scalar (by alternation of the uniform and nonuniform regions of the scalar substance) fields are not coincident. Therefore, in modeling turbulent flow, one must primarily allow for the effects of intermittence of both the dynamic and scalar fields.

On the Role of the PDF in Modeling of Turbulent Processes. When the processes of turbulent heat and mass transfer are modeled we need only have knowledge of the mean and "pulsation" characteristics (here single-point central moments) of dynamic and scalar fields. However in the case of turbulent combustion we should also know the distribution of the scalar substance, i.e., the PDF of this quantity.

The physicochemical processes of turbulent combustion are successfully modeled using the "conservative-scalar" method [5]. A characteristic feature of this method is that the characteristics of the field of the concentration z of a passive impurity with the PDF of the concentration of the passive impurity $P(z)$ are employed.[2] Thus, the method of "restored fuel concentration" making it possible to substantially improve the efficiency of modeling of turbulent-combustion processes has been developed in [2]. The essence of the method is that one uses the values of concentration of the basic chemical reactants, which are conditionally averaged over the turbulent medium $\langle c_k\rangle_{\mathrm{t}} = \int c_k(z)P_{\mathrm{t}}(z)dz$ and

[1]In modeling the energy-containing (large-scale) structure of nonuniform turbulent flow, it is the practice to call the intermittence "external." The intermittence phenomenon is one integral property of such flows. The phenomenon of "external" intermittence was first found by R. H. Corsin (see, e.g., [2]).

[2]With a certain normalization the quantity z is treated as the parameter of mixing of the fuel and the oxidant. In the case of combustion it is related to the concentration of the chemical reactants $c_k = c_k(z)$.

are related to the condition $\langle c_k \rangle = \int c_k(z)P(z)dz$ by the statistical relation $\langle c_k \rangle_t = \langle c_k \rangle / \gamma$. The method of calculation of the intermittence coefficient is also required.

The method of [2] now seems the most efficient, physically substantiated (within the framework of certain approximations), and, importantly, promising in terms of its further development. For example, in [3], the "conservative-scalar" method is constructed by modeling the conditional mean of the chemical reactants $\langle c_k | z \rangle$. Just as $\langle c_k \rangle_t$, the variable $\langle c_k | z \rangle$ ensures a more efficient description of combustion processes than the total mean $\langle c \rangle$. It is remarkable that, as in the method of [2], the system of differential equations of [3] obtained with the joint probability density $P(c_k, z)$ contains the equation to be solved for $P(z)$.[3] In other words, as in the method of [2], the turbulent liquid is identified by the concentration field of the passive impurity. An approach to modelling of the turbulent transfer of a scalar substance, which allows for the intermittence of both the dynamic and scalar fields of turbulent flow, is proposed in [6, 7]. In this approach, one uses the conditional PDFs $P_t(z)$ and $P_s(z)$, i.e., considers not only the flow field of the turbulent liquid but also the field of a nonuniform concentration in the turbulent medium itself. In view of the special role of the function $P(z)$ (and hence the functions $P_t(z)$ and $P_s(z)$), methods of its determination belong to the "PDF methods."

Method of the PDF of the Concentration of a Passive Impurity. There are, at least, two reasons why it is necessary to develop the method of the PDF of the concentration z of a passive impurity. First, no solution of the existing equation for the function $P(z) \equiv P(z; \mathbf{x}, t)$ in the entire region of the flow in question has been found to date.[4] The same is true of the problem of solution of the equation for the conditional PDF $P_t(z) = P(z|J = 1)$ with indicator of the turbulent medium J; this PDF provides a basis for the method of modeling of combustion processes in the turbulent medium of flow [6, 7].[5] (The form of the truncated function $P_t(z)$ in the turbulent medium of flow has been proposed, e.g., in [8, 9]). Second, and this should specifically be stressed, the issue of improving the accuracy of calculation of the parameters of the equation for the PDF, i.e., of such statistical characteristics as, e.g., $\langle z \rangle$ and $\langle z'^2 \rangle$ (or $\langle z \rangle_t$ and $\langle z'^2 \rangle_t$ in the case where the function $P_t(z)$ is used) has not been solved as yet. High requirements on the accuracy of calculation of the processes of transfer of a passive impurity in the case of turbulent combustion are determined by the strong nonlinear dependence of $c_k = c_k(z)$. Such a dependence creates conditions where small errors in the calculation of the statistical characteristics of a passive impurity lead to significant errors in the calculation of the sought concentrations of the chemical reactants c_k.

It follows from what has been said above that in the statistical hydrodynamics of turbulent flows, there is the problem on improving the efficiency of modelling of the characteristics of nonuniform temperature and concentration fields of the transferred substance. Solution of this problem primarily requires the development of the theory of turbulent transfer of a passive impurity under the conditions of intermittence of different dynamic and scalar fields.

Development of the Theory of Turbulent Heat and Mass Transfer. We present the concept of the mechanism of turbulent transfer of a certain scalar substance as a dynamically passive impurity in a turbulent flow: the turbulent transfer of a dynamically passive impurity is by convective turbulent diffusion, is gradient in character, and occurs only in the flow regions in which the turbulent liquid contains the impurity with a concentration z from the range $0 < z < 1$ (in what follows simply the intersection region $\Omega_s = \Omega_t \cap \Omega_z$); the turbulent transfer of momentum, heat, and substance in the region of intersection of the dynamic and scalar fields Ω_s is identical in character.[6]

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[3]In [3], the model of turbulent transfer of the reactive impurity is based on the equations for conditional mean $\langle c|z \rangle$ values of the concentration of a chemically reactive impurity c, which are averaged for a prescribed value of the concentration of the passive impurity z. The statistical characteristics of the field of the concentration z here are assumed to be a priori known.

[4]The difficulties are associated with both the problem of closure and the complicated character of the differential equation for $P(z)$. There are only solutions for the individual regions of turbulent flow after fairly strong simplifications due to the special hydrodynamic features of shear flow. The form of $P(z)$ is often simply prescribed.

[5]The essence of this method is that the processes of turbulent combustion are modeled only in the turbulent medium of the flow in question.

[6]This is essentially L. Prandtl's idea in his momentum-transport theory but now it is used only in the intersection region $\Omega_s \subset \Omega_t$. Here the "identity" of transfer is meant in a "narrow sense" [4] and is expressed by the equality of the coefficients $\nu_s = a_s$ which are involved in the conditionally averaged (over the region Ω_s) equations with a subscript s. These equations are coincident with (1) and (2) in form.

The adopted concept is based on the experimentally established fact that in the flow regions with a turbulent liquid Ω_t, there are regions with a uniform field of concentration of the passive impurity $z = 0$ and $z = 1$ in which the turbulent diffusion of the substance is absent. The mechanism of formation of such regions is attributed to the "entrainment" of the liquid from the external nonturbulent medium into turbulent flow followed by the turbulization of the liquid. A mathematical representation of the adopted concept are the relations

$$\langle z \rangle_s = \langle z \rangle_{t, z\in\Omega_s} ; \quad \langle z \rangle_s = \langle u \rangle_s / u_{max} . \tag{3}$$

Here $\langle z \rangle_s$ is the statistically averaged dimensionless (in mass fractions) concentration of the passive impurity, which is determined by the conditional sample of instantaneous values $z \in 0 < z < 1$ during the observation of the turbulent medium at a prescribed point of the flow, i.e., when the condition $(\mathbf{x}, t) \in \Omega_s$ is satisfied. In the intersection region Ω_s, the equality $\langle u \rangle_s \equiv \langle u \rangle_t$ holds; this follows from the obvious relation $\langle u \rangle_t \equiv \gamma_s \langle u \rangle_s + (1 - \gamma_s)\langle u \rangle_t$, where γ_s is the probability of observation of the nonuniform field of the scalar substance in the turbulent medium of flow.

A mathematical apparatus of the statistical theory of transfer of a scalar substance will be constructed as follows. We write the existing expression for the PDF of the concentration of a passive impurity as applied to the generalized (overall) flow region Ω consisting of the regions with turbulent and nonturbulent liquids:

$$P(z) = \gamma P_t(z) + \gamma_{n,z=1}\, \delta(z-1) + \gamma_{n,z=0}\, \delta(z) \quad \text{with the condition } \gamma + \gamma_{n,z=1} + \gamma_{n,z=0} = 1 . \tag{4}$$

An expression for the conditional PDF $P_t(z)$ in the flow's turbulent medium itself will be represented as

$$P_t(z) = \gamma_s P_s(z) + \gamma_{t,z=1}\, \delta(z-1) + \gamma_{t,z=0}\, \delta(z) \quad \text{with the condition } \gamma_s + \gamma_{t,z=1} + \gamma_{t,z=0} = 1 \tag{5}$$

on the basis that in the turbulent medium Ω_t itself, there can be regions (including individual unconnected ones) with the values $z = 0$ and $z = 1$. Then (4) and (5) yield

$$\langle z \rangle = \gamma \langle z \rangle_t + \gamma_{n,z=1} , \tag{6}$$

$$\langle z \rangle_t = \gamma_s \langle z \rangle_s + \gamma_{t,z=1} . \tag{7}$$

As is seen, to determine the total mean $\langle z \rangle$ we should primarily find γ_s and $\langle z \rangle_s$. The distribution of the values of the coefficient γ_s can be found by consideration of the flow field of the turbulent liquid with intermittence function J (indicator of the turbulent medium) and the nonuniform field of the impurity z from the range $0 < z < 1$ with intermittence function $J_z \equiv J_{z=\text{var}}$. We have $JJ_z = J_z$, since according to the adopted concept, the field of the turbulent liquid includes the nonuniform field of concentration of the passive impurity, whereas the intermittence function of the intersection field is $J_s = J_z | J = 1$. The interrelationship of the intermittence coefficients is determined by the total-probability formula $P(J{\cdot}J_z) = P(J){\cdot}P\,(J_z|J = 1)$ which, with the allowance for the notation $\langle J \rangle = \gamma$, $\langle J_z \rangle = \gamma_z$, and $\langle J_z \rangle_t = \gamma_s$, gives

$$\gamma_z = \gamma_s \gamma , \tag{8}$$

where the intermittence coefficient γ_z is the probability of observation of the nonuniform field of the scalar substance in the "generalized" flow field Ω. Now the expression for the total mean values of concentration (6) in the most general case of turbulent flow takes, with account for (7), the following form:

$$\langle z \rangle = \gamma_z (\langle z \rangle_s - 1) + \gamma (1 - \gamma_{t,z=0}) + \gamma_{n,z=1} . \tag{9}$$

The resulting equation (9) relates the total mean scalar characteristics to their conditional means through the intermittence coefficients γ and γ_z: the probabilities of observation of the turbulent liquid and the values of the concentration z from the range $0 < z < 1$ throughout the flow field Ω. Also, it contains the coefficients $\gamma_{t,z=0}$ and $\gamma_{t,z=1}$, i.e., the probabilities of observation of the values $z = 0$ and 1 in the turbulent medium Ω_t. Relation (9) is taken as the basis in modelling of unconditionally averaged characteristics of turbulent heat and mass transfer.

The developed propositions of the statistical theory of heat and mass transfer under intermittence conditions make it possible to formulate a new principle of construction of mathematical models of turbulent flows with a nonuniform field of a scalar substance.

Principle of Construction of Mathematical Models. The principle of construction of mathematical models of turbulent flows with a nonuniform field of a scalar substance, which allows for the effects of intermittence of different dynamic and scalar fields, is as follows. First we construct a mathematical differential model for conditional mean characteristics in the region of intersection of the dynamic and scalar fields Ω_s. The equations of the model are derived by averaging of the initial hydrodynamic equations using the conditional PDF (CPDF). The procedure of conditional averaging of the equations has been presented in [10, 11]. Then the system of equations averaged in this manner is closed using the corresponding hypotheses, "closure hypotheses." The intermittence coefficients γ and γ_z can be calculated with the approximate approaches of [2, 8]. In the final step, we use relation (9) and find the total mean of the values of the scalar impurity in the entire region of the flow in question. The profile of the mean velocity values is calculated from the existing formula

$$\langle u \rangle = \gamma \langle u \rangle_t + (1 - \gamma) \langle u \rangle_n , \tag{10}$$

in which the conditional mean velocities $\langle u \rangle_t$ and $\langle u \rangle_n$ are determined according to the method of [10, 11].

The basic difference of the proposed principle (method) of construction of mathematical models is that it allows for the process of intermittence not only of the regions with a turbulent liquid throughout the field of the flow in question but also the process of intermittence of the regions with a constant value of the scalar substance in the turbulent medium itself. The presence of such regions influences the result of the averaging of the transferred substance and leads to the fact that the total mean characteristics of the dynamic and scalar fields of turbulent flow markedly differ. The above discussion is confirmed by the existing experimental data. Of them, the most characteristic are the data [2] demonstrating the correctness of both the inequality $\gamma \neq \gamma_z$ and the fact (underlying the adopted concept) that the regions of flow with the greatest shear, i.e., where pressure pulsations are particularly strong, show the strongest difference in the characteristics of the dynamic and scalar fields.

Evaluation Testing of the Method. The proposed method of modeling is conveniently tested on jet-type turbulent flows. For such flows, the concentration of the impurity in the ambient nonturbulent medium is $z_n = 0$; in the potential core of the jet itself, it is $z_0 = 1$ (in the case of nonisothermal flow the dimensionless temperature $\langle \theta \rangle$ is selected accordingly). As is well known, behind the initial portion of such a jet (when the potential jet core disappears), in its self-similar region, the probability of observation of the concentration values $z_0 = 1$ is low, i.e., we have $\gamma_{n,z=1} \cong 0$. This simplifies the problem of modeling, since now, according to (6), we have

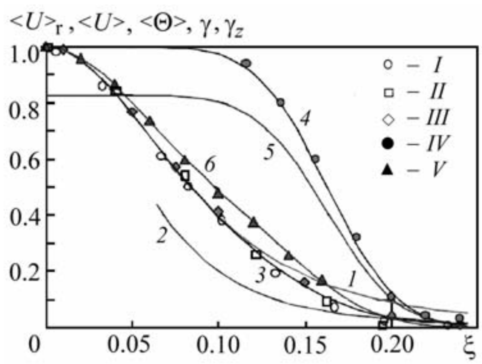
$$\langle z \rangle = \gamma \langle z \rangle_t . \tag{11}$$

Moreover, the probability of observation of the concentration with $z = 0$ in the turbulent medium itself of the flow in question is not very high; it can be assumed without a large error that $\gamma_{t,z=0} << 1$ in expression (9) (in condition (5), the value $\gamma_{t,z=0}$ should not be disregarded, since it can appreciably influence the value of other probabilities involved in this condition). Then expression (9) takes the form

$$\langle z \rangle = \gamma + \gamma_z (\langle z \rangle_s - 1) . \tag{12}$$

The resulting relations for the concentration of the passive impurity are also true of the temperature, when its differences are minor. We note, however, that expression (11) can be unacceptable for other types of flow. For example, we cannot similarly simplify expression (9) for flow in the mixing zone of two cocurrent flows of a liquid with concentrations $z = 0$ and $z = 1$, since in this case we have $\gamma_{n,z=1} \neq 0$.

Calculation results. Expression (12) was used to calculate the total mean values of the temperature and concentration of the passive impurity in a submerged axisymmetric jet, including a weakly heated one. The results of the performed calculations [12–14] of the conditional mean velocities (with the subscript r) and the total (with no sub-



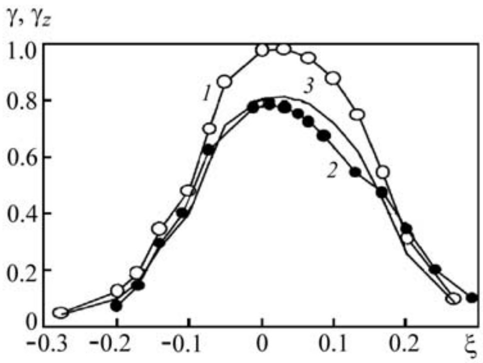


Fig. 1. Universal profiles of the conditional mean longitudinal velocities $\langle U\rangle_r$ (1 and 2), total mean longitudinal velocity $\langle U\rangle$ (3), intermittence coefficients γ and γ_z (4 and 5), and total mean temperature $\langle\Theta\rangle$ (6) in the self-similar region of the submerged axisymmetric heated jet. Experimental data: I) [12]; II) [13]; III, [14] for $\langle U\rangle$; IV, [4, 15] for γ; V, [4, 15] for $\langle\Theta\rangle$.

Fig. 2. Distributions of the values of the coefficients of intermittence of the turbulent liquid and the nonuniform temperature field in the mixing zone on the initial portion of the submerged heated axisymmetric jet: 1) γ, 2) γ_z, and 3) $\gamma_z = 0.825\gamma$. Dark and light symbols connected by the lines, experimental data [2].

script) mean velocity $\langle U\rangle_r = \langle u\rangle_r/u_{\max}$, and also of the total mean temperature $\langle\theta\rangle = (\langle T\rangle - T_{\min})/(T_{\max} - T_{\min})$ in the self-similar region of the jet, where $u_{\max}$ and $T_{\max}$ are the values of the velocity of the flow and its temperature on the jet's axis, are presented in Fig. 1. The formula for calculation of curve 4 describing the intermittence coefficient γ was selected according to the experimental data presented in this figure.[7]

The agreement between the calculations of the profile of the unconditionally averaged temperature (curve 6) and the experimental data makes it possible to infer that the characteristics of flow in the selected intersection region Ω_s are the most suitable characteristics for modeling of turbulent transfer. The use of these precisely characteristics enabled us to calculate the total (unconditional) mean values of temperature and concentration with a high degree of accuracy.

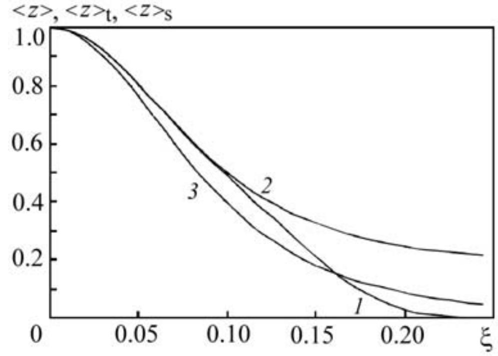
We note that the interrelationship of different intermittence coefficients involved in (12) was determined by the relation $\gamma_s = \gamma_z/\gamma \cong \text{const}$. The temperature profile was calculated with the value of the intermittence coefficient of the temperature field in the turbulent medium $\gamma_s = 0.825$, i.e., the intermittence coefficient γ_z (curve 5) was calculated from the formula $\gamma_z = 0.825\gamma$. Such a formula is substantiated by the calculations of γ_z values whose comparison to experimental data [2] shows quite a satisfactory agreement (curve 3, Fig. 2).

Calculations of the total and different conditional (for the fields Ω_t and Ω_s) mean values of the concentration of the passive impurity of the flow in question are presented in Fig. 3 and point to their maked difference.

Calculations of the concentration profile for the vertical submerged axisymmetric jet (Fig. 4) seem the most interesting. A certain deviation of experimental data [15] from calculated ones on the jet's edge can be due to both the difficulties of experimental determination of the "threshold" of the turbulent medium and the fact that the turbulent medium, in these experiments, was determined from the concentration field of the impurity.

Good agreement between the performed calculations and the known experimental data presented in Figs. 1–4 confirms the developed propositions of the statistical theory of turbulent transfer of a scalar substance.

[7]For calculation of γ we could, of course, use methods proposed, e.g., in [2] or [8]. But such methods are only approximate and their use would have led to a diminished purity of the carried-out evaluation testing. In other words, there has been no accurate and substantiated method of calculation of the intermittence coefficient γ to date. Therefore, we prescribed the distribution of the values of the intermittence coefficient according to experimental data for the highest purity of evaluation testing.



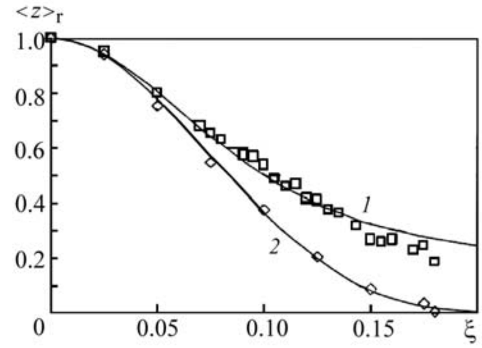


Fig. 3. Calculated profiles of the total and conditional mean values of the concentration of the passive impurity under the conditions corresponding to Fig. 1: 1) $\langle z \rangle$, 2) $\langle z \rangle_t$, and 3) $\langle z \rangle_s$.

Fig. 4. Radial profiles of the conditional and total concentration of the impurity in the vertical axisymmetric submerged jet: 1) $\langle z \rangle_t$ and 2) $\langle z \rangle$. Symbols, experimental data [16].

Equation for the CPDF of the Concentration of the Passive Impurity. The developed theory enables us to address ourselves to the development of a PDF method with the aim of deriving new equations for CPDFs. The need for such equations is determined by the following circumstance.

The equation for the single-point PDF of scalar quantities of a turbulent flow is well known [2]:

$$\frac{\partial \rho P(c)}{\partial t} + \nabla \left(\rho \int \mathbf{u} P(\mathbf{u}, c)\, d^3\mathbf{u} \right) = -\gamma \frac{\partial^2}{\partial c^2} \rho \langle N \rangle_{t,c} P_t(c) - \gamma \frac{\partial}{\partial c} \rho W P_t(c)\,. \tag{13}$$

Here c is the variable which characterizes one of the three regimes of flow depending on the type of the considered problem: $c = z$ in mixing of chemically inert gases, when z is the concentration of the passive impurity, and in diffusion combustion of the nonpremixed reactants of the fuel and the oxidant, when z is treated as the "restored concentration of the fuel;" $c = (T - T^{(o)})/(T^{(b)} - T^{(o)})$ is the dimensionless value of temperature in combustion of the premixed fuel mixture.

As is well known [2], Eq. (13) after its closure contains the total and conditional mean characteristics of flow. Moreover, it contains the intermittence coefficient of the turbulent liquid, whose determination is a fairly difficult and independent problem. All this considerably complicates solution of this equation and hinders the development of a more efficient method of modeling of the processes of turbulent mixing. The problem can be greatly simplified, if we derive an equation for the conditional probability density, i.e., obtain an equation for the CPDF of the concentration of the passive impurity in a nonuniform intersection field Ω_s. Derivation of such an equation and its closure and solution for the region of strongly intermittent flow are of great interest for modeling of the processes of turbulent combustion.

The procedure of derivation of an equation for the CPDF of a certain scalar quantity in the intersection field Ω_s is analogous to [2]. The use of the properties of characteristic functions and Fourier transformation makes it possible to obtain such an equation in the following general form [16]:

$$\frac{\partial \rho P_s(c)}{\partial t} + \nabla \left(\rho \langle u \rangle_{s,c} P_s(c) \right) = -\frac{\partial^2}{\partial c^2} \rho \langle N \rangle_{s,c} P_s(c) - \frac{\partial}{\partial c} \rho W P_s(c)\,. \tag{14}$$

A fundamental difference of the obtained equation is that it describes processes occurring in the regions with a turbulent liquid nonuniform in concentration. For this reason, Eq. (14) contains no intermittence coefficients. Also, it is important that, unlike (13), it involves only one sought probability density $P_s(c)$. Equation (14) is closed with the hypotheses making it possible to express individual terms of the equation by the sought hydrodynamic quantities. The main difficulty is determination of "double" conditional mean velocity $\langle \mathbf{u} \rangle_{s,c}$ and scalar dissipation $\langle N \rangle_{s,c}$, i.e., modeling proper of the processes of convective diffusion and mixing of the reactants to the molecular level. The problem of modeling varies depending on the considered regime of flow and becomes different in character. This problem is solved most simply in the case $c = z$, i.e., in the case of just turbulent mixing or diffusion turbulent combustion where the method with the "restored concentration of the fuel" is used [2]. In this case, according to [16], we have

$$\frac{\partial \rho P_s(z)}{\partial t} + \nabla (\rho \langle u \rangle_{s,z} P_s(z)) = -\frac{\partial^2}{\partial z^2} \rho \langle N \rangle_{s,z} P_s(z) , \tag{15}$$

on the basis of the adopted concept, we take the expression

$$\langle \mathbf{u} \rangle_{s,z} = \langle \mathbf{u} \rangle_s + q_s \sigma_s^{-2} (z - \langle z \rangle_s) ,$$

which appears more convincing than the analogous expression in [2]. Here we have

$$q_s = \langle (u - \langle u \rangle_s) (z - \langle z \rangle_s) \rangle_s , \quad \sigma_s^2 = \langle (z - \langle z \rangle_s)^2 \rangle_s .$$

The term describing the process of mixing to the molecular level can be transformed as

$$\frac{\partial^2}{\partial z^2} \rho \langle N \rangle_{s,z} P_s(z) = \langle N \rangle_s \frac{\partial^2}{\partial z^2} \rho P_s(z) , \quad \langle N \rangle_{s,z} = \langle N \rangle_s$$

in view of the well-known Kolmogorov–Obukhov hypothesis for the independence of the N and z fields. This is in agreement with the results of development of the theory of a small-scale turbulence structure under intermittence conditions [17, 18]. As a consequence of the performed closure and in the case of a statistically stationary process Eq. (14) takes the form

$$\frac{\partial^2}{\partial z^2} P_s(z) + \frac{\langle \mathbf{u} \rangle_s}{\langle N \rangle_{s,z}} \nabla P_s(z) + \frac{\nabla [q_s \sigma_s^{-2} (z - \langle z \rangle_s) P_s(z)]}{\langle N \rangle_{s,z}} = 0 . \tag{16}$$

Our next problem is solution of this equation.

Solution of the Equation for the CPDF on the Edge of the Turbulent Jet. This problem is most simply solved for the edge of the turbulent jet. In the case, by analogy with [2], we obtain the equation

$$\frac{\partial^2 P_s(z)}{dz^2} + (a_1 - b_1 z) P_s(z) = 0 \tag{17}$$

with boundary conditions

$$P_s(z) \to 0 \quad \text{at} \quad z \to 0 , \quad P_s(z) \to 0 \quad \text{at} \quad z \to \infty . \tag{18}$$

Equation (17) has the exact solution only at certain values of a_1 and b_1 [2]; in our case this solution has the following form:

$$P_s(z) = R_1 b_1^{1/3} \,\mathrm{Ai}(\chi) , \quad \text{where} \quad \mathrm{Ai}(\chi) = \frac{1}{\pi} \int_0^\infty \cos \left(\frac{t^3}{3} + t\chi \right) dt , \quad \chi = b_1^{1/3} \left(z - \frac{a_1}{b_1} \right). \tag{19}$$

In self-similar form, the obtained solution (19) is represented by the expressions

$$P_s(z) = \frac{p_s(\zeta)}{\langle z \rangle_s} , \quad p_s(\zeta) = R \,\mathrm{Ai}(\chi) , \quad \chi = b^{1/3} \zeta + v_1 , \quad \zeta = \frac{z}{\langle z \rangle_s} . \tag{20}$$

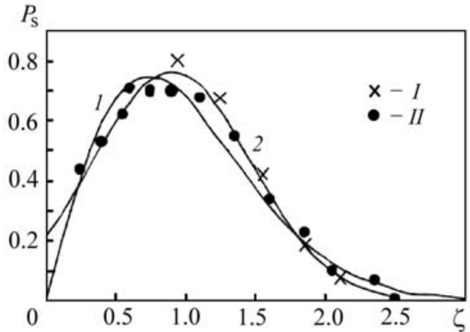


Fig. 5. Conditional PDF $p_s(\zeta)$ of the concentration of a passive impurity for the region Ω_s. Calculations have been performed for the edge of a submerged axisymmetric methane jet in air: 1) from the solution (20) and 2) from formula (21). Symbols, experimental data: I) [20] and II) [21].

The performed calculations give the parametric values $R = R_1 b^{1/3} = 1.403$ and $b^{1/3} = b_1^{1/3}\langle z\rangle_s = 1.788$. The value of the root $v_1 = a_1 b_1^{-2/3} = -2.338$ is found by calculations of the Airy function. The variance $S_k = \sigma_s/\langle z\rangle_s$ os the concentration of the passive impurity is determined by the expression $S_k = \int_0^4 (\zeta - 1)^2 p_s(\zeta) d\zeta = 0.555$. The maximum of the $p_s(\zeta)$ curve is shifted by $D = 0.75$. Let us use these data for construction of a truncated PDF. We prescribe it in the form

$$P_s(z) = \frac{K}{\sqrt{2\pi}\, S_k \langle z\rangle_s} \exp\left[-\frac{\left(\frac{z}{\langle z\rangle_s} - D\right)^2}{2S_k^2}\right] \tag{21}$$

with three unknown parameters (S_k, K, and D). The developed program of numerical calculation has shown that formula (21) is in best agreement with experimental data when $D = 0.9$. The parametric values $S_k = 0.555$ and $K = 1.055$ (which was determined from the normalization condition $p_s(z)$ at the prescribed initial value of S) were selected from the exact solution (20) obtained for the jet's edge. The results of calculations of the normalized "accurate" and truncated functions $p_s(\zeta)$ are compared to experimental data in Fig. 5 and show quite an acceptable agreement.

Conclusions. It emerges that the "different mechanism" of turbulent transfer of momentum, heat, and substance is attributed to the presence of the intermittence of not coincident dynamic and scalar fields of turbulent flow. This precisely causes the difference in the profiles of the total mean values of velocity and temperature (concentration). The reasons for this difference are in the initial hydrodynamic equations and are mainly due to the nonlocal action of pressure pulsations on the structure of turbulent flow. The $\mathrm{Pr_t}$ and $\mathrm{Sc_t}$ numbers traditionally used in modeling can be treated as a certain artificial technique for taking account of the effect of intermittence of different fields.

The proposed method of modeling of the processes of turbulent heat and mass transfer makes it possible to construct more detailed and efficient models without introducing additional empirical parameters. Construction of turbulent-transfer models is reduced to the modeling of flow only in the regions Ω_s, i.e., where the flame front is located. Calculation of the total and conditional mean velocities of flow of a nonturbulent liquid is not necessary. This conclusion is very important, since the above possibility sharply reduces the volume of the modeling of turbulent-combustion processes and leads to a more efficient construction of the models. Clearly, the semiempirical character of the models is preserved and the mechanism of interaction of the turbulent and nonturbulent liquids is allowed for by the empirical parameter of downstream "expansion" of the turbulent medium.

The presented concept of the mechanism of turbulent heat and mass transfer is based on the existing and experimentally verified (e.g., in [2]) physical ideas of the pattern of turbulent flow. The rigorous mathematical apparatus of transport theory presented in this work is based on the novel approach in modeling of intermittent turbulent flows [10, 11]. Evaluation testing of the theory and method of modeling of turbulent flow with a nonuniform scalar field has been carried out on the basis of the existing experimental data. Good agreement between the calculations and experimental data is also evidence in favor of the method developed here. This is directly confirmed by the models constructed by the method of [10, 11] for the dynamic fields of different types of turbulent flows (see, e.g., [21] and others). Calculations of the conditional and total mean velocities, which have been performed from these models, virtually agree with the existing experimental data. Also, we emphasize that in this method, we exclude from consideration differential equations of turbulent transfer of heat and substance, i.e., equations of the (2) type. This is not surprising, since here consideration is given to the transfer of a dynamicaly passive substance which has no influence on the flow dynamics by definition.

NOTATION

Ai, Airy function; a_t, turbulent thermal diffusivity; c_k, concentration of chemical reactants; $f(\mathbf{x}, t)$, hydrodynamic quantity; $\langle f(\mathbf{x}, t)\rangle$, unconditional (total) mean of the hydrodynamic quantity; $f'(\mathbf{x}, t)$, fluctuations (pulsations) of the hydrodynamic quantity; $\langle f(\mathbf{x}, t)\rangle_r$, quantity conditionally averaged over a turbulent or nonturbulent medium (liquid); J and γ, function and coefficient of intermittence of the dynamic field of a turbulent liquid; J_z and γ_z, function and coefficient of intermittence of the nonuniform concentration field of a passive impurity; J_s and γ_s, function and coefficient of intermittence of the "intersection" field; $\langle N\rangle_{t,c}$, conditional mean of scalar dissipation; $P(f)$, PDF of f values; $P_r(f)$, conditional PDF of the f values of a turbulent or nonturbulent medium; $P(f, z)$, joint PDF; P, pressure; Sc, Schmidt number; Pr, Prandtl number; T, temperature; u and v, longitudinal and transverse components of the flow velocity; x, y, coordinates of a point; z, concentration of a passive impurity; δ, delta function; $\zeta = z/\langle z\rangle_s$; Θ, dimensionless temperature; ν_t, coefficient of turbulent viscosity; $\xi = x/y$; ρ, density of the medium; Ω, generalized region of hydrodynamic (dynamic and scalar) fields of the flow in question; Ω_t, region of hydrodynamic fields of a turbulent liquid; Ω_n, region of hydrodynamic fields of a nonturbulent liquid; Ω_z, region of the nonuniform concentration field of a passive impurity $z \in 0 < z < 1$; Ω_s, region of "intersection" of the dynamic fields of a turbulent liquid and the scalar fields of a transferred substance (concentration of a passive impurity), i.e., $\Omega_s = \Omega_t \cap \Omega_z$. Subscripts: t, turbulent liquid; n, nonturbulent liquid; r, takes value of t for a turbulent liquid and the value of n for a nonturbulent liquid; var, subscript characterizing a variable; k, chemical-reactant number.

REFERENCES

1. J. Lumley, B. Launder, and P. Bradshaw, Collaborative testing of turbulent models, *Trans. ASME*, *J. Fluid Eng.* (1996).
2. V. R. Kuznetsov and V. A. Sabel'nikov, *Turbulence and Combustion. Hemisphere* [in Russian], Nauka, Moscow (1986).
3. A. Y. Klimenko and R. W. Bilger, Conditional moment closure for turbulent combustion, *Prog. Energy Combust. Sci.*, **25**, 595–687 (1999).
4. J. O. Hinze, *Turbulence* [Russian translation], Izd. Fiz.-Mat. Lit., Moscow (1963).
5. P. A. Libby and F. A. Williams, *Turbulent Reacting Flows*, Academic Press, New York (1994).
6. Yu. V. Nuzhnov, On the theory of diffusion turbulent combustion. Transfer of a passive impurity in a turbulent flow, *Proc. II Int. Symp. on Combustion and Plasma Chemistry*, Almaty (2003), pp. 84–88.
7. Yu. V. Nuzhnov and Z. A. Mansurov, Development of the theory of turbulent heat- and mass transfer with particular emphasis on turbulent combustion, *Proc. VII Asia-Pacific Int. Symp. on Combustion and Energy Utilization*, December 15–17, 2004, Hong Kong SAR. CD A4-N193.
8. Yu. V. Nuzhnov and B. P. Ustimenko, *Diffusion Combustion of Turbulent Flows*, Republic of Kazakhstan, Nauka, Alma-Ata (1993).
9. Yu. V. Nuzhnov and B. P. Ustimenko, On the statistical theory of a diffusion turbulent jet, *Izv. SO Ross. Akad. Nauk*, *Fiz. Goreniya Vzryva*, **31**, No. 2, 41–46, Novosibirsk.

10. Yu. V. Nuzhnov, Conditional averaging of the Navier–Stokes equations and a new approach to modeling intermittent turbulent flows, *J. Fluid Dynam.*, **32**, No. 4, 489–494 (1997).
11. Yu. V. Nuzhnov, Method of "autonomous" modeling of turbulent flows under interruption conditions. Part 1. Statement of the problem, *Vestn. KazNU*, **60**, No. 1, 87–98 (2009).
12. I. C. Laurence, Intensity scale and spectra of turbulence in mixing region of free subsonic jet, NACA Report No. 1292 (1956).
13. S. Corrsin and A. L. Kistler, Free stream boundaries of turbulent flows, NACA Report No. 1244 (1955).
14. I. Wygnanski and H. Fiedler, Some measurements in the self-preserving jet, *J. Fluid Mech.*, **38**, Pt. 3, 577–612 (1969).
15. D. Papantoniou and E. List, Large-scale structure in the far field of buoyant jets, *J. Fluid Mech.*, **209**, 151–190 (1989).
16. Yu. V. Nuzhnov, Simulation of turbulent combustion on the basis of conditional pdfs of a conservative scalar, *Vestn. KazNU*, **46**, No. 3, 119–130 (2005).
17. Yu. V. Nuzhnov, On the development of the theory of small-scale turbulence with refinement of the Kolmogorov and Obukhov laws, *Vestn. KazNU*, *Ser. Khimicheskaya*, **20**, No. 3, 18–27 (2000).
18. Yu. V. Nuzhnov, The lognormal law of the space probability distribution of energy dissipation under the conditions of intermittence of turbulent flows, *Int. Symp. "Combustion and Plasmachemistry,"* IPG, Almaty (2001), pp. 45–48.
19. A. D. Birch, D. R. Brown, M. G. Dodson, and G. R. Tomson, The turbulent concentration field of a methane jet, *J. Fluid Mech.*, **88**, Pt. 3, 431–450 (1978).
20. I. Ebrahimi, R. Gunter, and F. Haberda, Wahrscheinlichkeitsdichteverteilungen der Konzentrazion in isothermen Luft-Freistrahlen, *Forsch. Ing.-Wes.*, **43**, 2, 47–52 (1977).
21. Yu. V. Nuzhnov and B. P. Ustimenko, A model of turbulent near-wall boundary layer under intermittence conditions, *Dokl. Nats. Akad. Nauk RK*, Parts 1, 2, No. 2, 18–24 (2001); No. 2, 17–25 (2002).