

## Necessary and Sufficient Conditions for Oscillations of Functional Differential Equations

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In this survey, necessary and sufficient conditions for the oscillation of solutions of retarded, advanced and neutral differential equations of first and higher order with one or several constant coefficients and constant arguments, in terms of the characteristic equation, are presented. Explicit (in terms of the constant coefficient and constant argument only) necessary and sufficient conditions are also presented in the case of one argument. In the case of  $n$ th order equations necessary and sufficient conditions for the oscillation of all solutions are presented when  $n$  is odd, while necessary and sufficient conditions for the oscillation of all bounded solutions are presented when  $n$  is even. In this case explicit sufficient conditions for the oscillation of all solutions are presented when  $n$  is odd, while explicit sufficient conditions for the oscillation of all bounded solutions for retarded equations and of all unbounded solutions for advanced equations are presented when  $n$  is even. In the case of several arguments explicit but sufficient conditions only are given and the results are also extended to equations with several variable coefficients.

**Key words:** Oscillation; Delay, Necessary and sufficient conditions, Characteristic equation, Difference Equations.