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"simpler" partial orders naturally defined for the structure. Indeed, it is known that every partial order is embeddable into the product order of a family of *linear* orders. The order dimension of a given partial order is defined as the least cardinality of such a family. Thus, the order dimension of a degree structure tells us how many linear orders at least we should have so that the degree structure is embeddable into the product order of those linear orders. The concept "order dimension" was introduced by Dushnik and Miller in 1941, and it is also called Dushnik-Miller dimension. As our main results on the order dimensions of degree structures, this talk includes the following results: the order dimension of Turing degree structure is uncountable and at most the cardinality of the continuum; the order dimension of Muchnik degree structure is the cardinality of the continuum; and the order dimension of Medvedev degree structure is lying between the cardinality of the continuum and the cardinality of the power set of the continuum.

▸ ASSYLBEK ISSAKHOV, *Ideals without minimal numberings in the Rogers semilattice.*
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It is well known many infinite families of c.e. sets whose Rogers semilattice contains an ideal without minimal elements, for instance, the family of all c.e. sets, [1]. Moreover, there exists a computable family of c.e. sets whose Rogers semilattice has no minimal elements at all, [2]. In opposite to the case of the families of c.e. sets, for every computable numbering α of an infinite family $\mathfrak{F}$ of computable functions, there is a Friedberg numbering of $\mathfrak{F}$ which is reducible to α, [1]. This means that the Rogers semilattice of any computable family of total functions from level 1 of the arithmetical hierarchy contains no ideal without minimal elements.

We study computable families of total functions of any level of the Kleene-Mostowski hierarchy above level 1 and try to find elementary properties of Rogers semilattices that are different from the properties of Rogers semilattices for the families of computable functions.

THEOREM 1. *For every n, there exists a Σ^0_{n+2}-computable family of total functions whose Rogers semilattice contains an ideal without minimal elements.*

Note that every Rogers semilattice of a Σ^0_{n+2}-computable family $\mathfrak{F}$ contains the least element if $\mathfrak{F}$ is finite, [1], and infinitely many minimal elements, otherwise, [3].

Theorem 1 is based on the following criterion that extends the criterion for minimal numbering from [2].

THEOREM 2. *Let α be a numbering of an arbitrary set S. Then there is no minimal numbering of S that is reducible to α if and only if, for every c.e. set W, if $\alpha(W) = S$ then there exists a c.e. set V such that $\alpha(V) = S$ and, for every positive equivalence ε, either $\varepsilon \restriction W \nsubseteq \theta_\alpha$ or $W \nsubseteq [V]_\varepsilon$.*

[1] YU. L. ERSHOV, ***Theory of numberings***, Nauka, Moscow, 1977 (in Russian).

[2] S. A. BADAEV, *On minimal enumerations*, ***Siberian Adv. Math.***, vol. 2 (1992), no. 1, pp. 1–30.

[3] S. A. BADAEV AND S. S. GONCHAROV, *Rogers semilattices of families of arithmetic sets*, ***Algebra and Logic***, vol. 40 (2001), no. 5, pp. 283–291.

▸ ALEKSANDER IVANOV, *Extreme amenability of precompact expansions of countably categorical structures.*