LOGIC COLLOQUIUM 2013
ÉVORA, PORTUGAL
JULY 22–27

LOGIC COLLOQUIUM 2013
ÉVORA - PORTUGAL

Scientific Program
&
Abstracts

Session 4 room 119

► THEODORE A. SLAMAN AND ANDREA SORBI, *Downwards density and incomparability in initial segments of the enumeration degrees.*
Department of Mathematics, The University of California, Berkeley, CA 94720-3840 USA.
E-mail: `slaman@math.berkeley.edu`.
Dipartimento di Ingegneria dell'Informazione e Scienze Matematiche, Università di Siena, 53100 Siena, Italy.
E-mail: `sorbi@unisi.it`.

It is known that there is no minimal enumeration degree. We improve on this result by showing:

THEOREM 1. *Below every nonzero enumeration degree one can embed every countable partial order.*

The result is in fact a particular case of the following:

THEOREM 2. *If* $\mathbf{a}, \mathbf{b}$ *are enumeration degrees, with* $\mathbf{a}$ *total, and* $\mathbf{a} < \mathbf{b}$, *then in the degree interval* $(\mathbf{a}, \mathbf{b})$, *one can embed every countable partial order.*

Session 4 room 119

► ASSYLBEK ISSAKHOV, *Computable numberings of the families of total functions in the arithmetical hierarchy.*
Department of Mechanics and Mathematics, Al-Farabi Kazakh National University, 71 Al-Farabi Ave., Almaty 050038, Kazakhstan.
E-mail: `asylissakhov@mail.ru`.

A numbering $\nu : \omega \mapsto \mathcal{F}$ of a family of unary computable functions is called computable if the binary function $\nu(n)(x)$ is computable, [1]. A Friedberg numbering of a family is just a computable one-to-one numbering. It is well-known that the Rogers semilattice of a computable family $\mathcal{F}$ either consists of one element or is infinite, [1]; and that, in the non-trivial case, it is never a lattice and has no maximal elements; and contains either one or infinitely many minimal elements, [2].

We generalize the notion of computable numbering for the families of functions in the arithmetical hierarchy following [3]. Let $\mathcal{F}$ be a family of total unary functions from Σ^0_{n+1}, $n \in \omega$. A numbering $\nu : \omega \mapsto \mathcal{F}$ is called Σ^0_{n+1}-computable if the binary function $\nu(n)(x)$ is computable relative to the oracle $\emptyset^{(n)}$ [3].

THEOREM 1. *Let* $\mathcal{F} \subseteq \Sigma^0_{n+2}$ *be an infinite* Σ^0_{n+2}*-computable family of total functions. Then* $\mathcal{F}$ *has infinitely many pairwise non-equivalent Friedberg numberings.*

THEOREM 2. *There are a family* $\mathcal{F}$ *and* Σ^0_{n+2}*-computable numbering* α *of the family* $\mathcal{F}$ *such that no Friedberg numbering of* $\mathcal{F}$ *is reducible to* α.

This is a solution to Question 2, [4]:

THEOREM 3. *If* $\mathcal{F}$ *contain at least two functions, then* $\mathcal{F}$ *has no principal* Σ^0_{n+2}*-computable numbering.*

[1] YU. L. ERSHOV, ***Theory of numberings***, Nauka, Moscow, 1977 (in Russian).

[2] S. S. MARCHENKOV, *The computable enumerations of families of general recursive functions*, ***Algebra and Logic***, vol. 11 (1972), no. 5, pp. 326–336.

[3] S. A. BADAEV AND S. S. GONCHAROV, *Rogers semilattices of families of arithmetic sets*, ***Algebra and Logic***, vol. 40 (2001), no. 5, pp. 283–291.

[4] S. A. BADAEV AND S. S. GONCHAROV, *The theory of numberings: Open problems*, ***Contemporary Mathematics*** (University of Colorado, Boulder), (Peter A. Cholak, Steffen Lempp, Manuel Lerman and Richard A. Shore, editors), vol. 257, American Mathematical Society, 2000, pp. 23–38.