Calculation of Friction Resistance of End Plates Affecting Flat Jet Fading

Scientific Research Institute of Experimental and Theoretical Physics,
Al-Farabi Kazakh National University, Almaty, Kazakhstan

Abstract: This research shows results of calculation of friction resistance of end plates affecting tendencies of free flat jet behavior. Flow diagram of jet flowing among end surfaces is built up. Estimate of friction at turbulent boundary layer is fulfilled. Calculation formula for the first study which describes variation of maximum jet velocity at a first approximation is acquired. These calculation data are compared with the experimental data. Also, a theoretical calculation of the second study was conducted for this study calculation formula about the maximum velocity variation was received.

Key words: Plates, affecting, flow diagram, calculation formula, experimental data, theoretical, calculation

INTRODUCTION

During recent decades dynamic and pulsation characteristics of free three-dimensional jet flowing out of the nozzle with rectangular outlet section within the major and partially the initial sections of friction have been studied in details (Abramovich et al., 1984; Quinn, 1992; Faghani et al., 2010). During recent time also vortex structure has been studied and its effect on development of turbulent and averaged flow parameters within the initial, transitional and major sections of free jet stream have been focused. At studying flat jet in experimental plants as a rule in order to exclude interference of finiteness of rectangular nozzle height the flow field is confined by end plates installed in parallel with the flow direction as continuation of end walls of outlet section of the rectangular nozzle. Here as we can see by virtue of influence of end walls, we have a flat jet confined by these side walls instead of three dimensional jet. We might as well say that new obtained experimental and theoretical data provide wide information on effect of end walls and large scale coherent vortexes on development of turbulent jets flowing out of the rectangular nozzle. For instance in the research (Isataev et al., 2015) friction resistance of end plates affecting tendencies of free flat jet behavior is studied by experiments. During recent time coherent flow structures of walls jets are also under focus of attention (Namgyal and Hall, 2013). This investigation area is an important subject for studies. Also, it’s important to carry on studies of dynamic flow parameters. This research study described as continuation of experimental studies shown in the research study (Isataev et al., 2015) contains theoretical calculation of friction resistance of end plates affecting tendencies of free flat jet behavior.

CALCULATION OF RESISTANCE INFLUENCE ALONG END WALLS

To build up calculation of resistance influence of end walls on flat jet fading let’s consider the following diagram of jet flowing among end surfaces. Figure 1 shows diagrams of a jet stream confined by flat end walls in planes xoy and xo. In the plane xoy, the jet just as in the ordinary free jet has initial section (index “I”), transitional section (index “T”) and major section as well as side free shifting borders, the nozzle width along the axis oy equals 2b. In the plane xo, the jet flowing out of the nozzle with height 2h on sides along the axis oz is confined by end plates. Within the first section of the jet after leaving the nozzle along the end walls laminar or turbulent boundary layers are developing with homogenous profile along the axis z between the limits boundary layers. Development of boundary layers is analogical to the boundary layer typical for homogeneous flow flowing along the plate. At the end of the 1st section boundary layers join on the jet axis and the 2nd jet section begins in which in the plane oz the flow is analogous to current flow in a flat channel. Accordingly development of the boundary layer and the current within the 1st section are analogical to homogeneous flow flowing along the plate within the 2nd section-analogical to current flow in a flat channel.
Geometrical parameter \( \lambda = 2h/2b \) characterizes relative elongation of the exhaust nozzle area. Subject to the foregoing within the 1st jet section let's assume variation of boundary layer thickness along \( z \) on the end plates in the form of the flowing dependencies:

\[
\delta_z = \frac{5.0 \cdot x}{\frac{U_m x}{v}} \tag{1}
\]

For laminar boundary layer:

\[
\delta_y = \frac{0.37 x}{\left( \frac{U_m x}{v} \right)^{0.5}} \tag{2}
\]

For turbulent boundary layer: Here \( x \)-longitudinal coordinate, \( U_m \)-velocity along the jet axis, \( v \)-kinematical viscosity and \( U_m x/v \)-let's assume it as Reynolds number \( Re_m = U_m x/v \). Length of the first section is defined based on the condition \( x = x_i \) at \( \delta_z = h \). Accordingly to calculate wall resistance we can use the formulas from the research study (Isataev et al., 2015):

\[
C_\zeta = \frac{0.664}{\sqrt{Re_m}} \text{ or } C_\zeta = \frac{0.0576}{\left( \frac{U_m x}{v} \right)^{0.2}}
\]

After joining the boundary layers resistance law in a flat channel is applicable for the second flow section with hydraulic resistance coefficient \( \zeta \) for the laminar flow:

\[
\zeta = \frac{16}{Re} \quad \text{where} \quad Re = \frac{U_m 2h}{v} \tag{3}
\]

For turbulent flow:

\[
\zeta = \frac{0.3164}{Re^{0.4}} \quad \text{where} \quad Re = \frac{U_m d_i}{v} \tag{4}
\]

where, \( d_i = 4F/\pi \)-hydraulic diameter defined as ratio of quadruplicated section area of channel \( F \) to its perimeter \( \pi \).

There is an approximated calculation of variation of total impulse of the jet affected by resistance of end walls for both sections under question under turbulent conditions of jet flow provided below.

**CALCULATION OF RESISTANCE AT TURBULENT BOUNDARY LAYER**

In presence of resistance of end walls the total jet impulse is not preserved and will reduce along the jet length:

\[
\frac{dK}{dx} = -2 \int_{-\delta_z}^{\delta_z} \tau_w dy \tag{5}
\]

Where:

\( K \) = Current total impulse in an optional jet section

\( \tau_w \) = Friction stress at the wall at distance \( y \) from the symmetry plane

\( \delta_z \) = Total jet halfwidth equaling to the distance from the axis to the outside boundary at \( U = 0 \)

As it's shown in the Fig. 1, there are boundary layers on end walls with \( \delta_z \) thickness in the first section of the jet stream and the central section with constant velocity \( U_m \) in the section \( y = 0 \). Let's assume that the jet width along the axis \( y \) is not changing along the axis \( z \) and equals to \( \delta_z \). Let's take velocity distribution across the section in the form of polynom (offered by G. Schlichting) in which \( U_m \) in the central part along the axis \( z \) won't change. Let's take velocity change in the wall-wise area in the form of power law as follows:

\[
\frac{U}{U_m} = \left( 1 - \frac{z}{\delta_z} \right)^{1/3} \tag{6}
\]

Where:

\[
\frac{U_1}{U_m} = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4 \tag{7}
\]

here \( U \)-longitudinal component of velocity, \( U_1 \)-velocity at the edge of the wall-wise boundary layer at the distance \( \delta_z \) of the wall with corresponding distances \( \eta = y/\delta_z \) from the plane \( z = 0 \) in the given section (\( \delta_z \)-total jet halfwidth
equaling to the distance from the external limit at \( U = 0 \) and related to the conditional width \( \delta_3 = 2.59 \delta \) for numbers \( \text{Re} = U_m 2h/\nu <10 \), \( n = 7 \) and \( <U> = 0.817 U_m \). Inserting Eq. 7 in 6 we will acquire velocity distribution in the wall-wise boundary layer:

\[
\frac{U}{U_m} = \left( 1 - \frac{x}{\delta_3} \right)^{\frac{3}{7}} \left( 1 - \frac{6 \nu}{\delta^2} + \frac{8 \nu^2}{\delta^3} \right) \tag{8}
\]

Then total jet impulse in the section being at the distance \( x \) away from the nozzle will equal:

\[
K = \int_0^\infty \rho U^2 \mathrm{d}z = 2 \int_0^x \rho U^2 \mathrm{d}y + 2 (h - \delta_3) \int_0^\infty \rho U^2 \mathrm{d}y \tag{9}
\]

where, \( \rho \)-density of liquid (gas). Taking into account (Eq. 7 and 8):

\[
K = 4 \rho U_m^2 \delta_3 \int_0^x \left( 1 - \frac{z}{\delta_3} \right)^{\frac{3}{7}} \left( 1 - \frac{6 \nu}{\delta^2} + \frac{8 \nu^2}{\delta^3} \right) \mathrm{d}z +

2 (h - \delta_3) \rho U_m^2 \int_0^\infty \left( 1 - \frac{6 \nu}{\delta^2} + \frac{8 \nu^2}{\delta^3} \right) \mathrm{d}z \tag{10}
\]

After computing the values of integrals, on rearrangement we have:

\[
K = \rho U_m^2 \delta_3 \left( \frac{20}{63} \delta_3 + \frac{4}{7} h \right) \tag{11}
\]

As the measurements show (Isaev et al., 2015), distributions of friction stress on the end walls in the coordinates \( z/\delta_3 = \Gamma(y/\delta) \) are similar to the velocity profile in the major section and summarized friction stress applied on the section with dimensions \( 4 \delta_3 \) \( \mathrm{dx} \) on both end walls will be equal:

\[
2 \int_0^{\delta_3} \tau_{\nu} \mathrm{d}y = 4 \tau_{\nu} \delta_3 \int_0^\infty \left( 1 - \frac{6 \nu}{\delta^2} + \frac{8 \nu^2}{\delta^3} \right) \mathrm{d}z = 4.15 \delta_3 \tau_{\nu} \mathrm{dx} \tag{12}
\]

Where:

\( \tau_{\nu} \) = Friction stress on the wall at the distance \( y \) from the symmetry plane
\( \tau_{\nu m} \) = Maximum friction stress on the wall at \( y = 0 \)

Inserting the values \( \delta_3, \delta, \delta_0, K_\nu, \tau_{\nu m} \) to Eq. 5, we obtain:

\[
\frac{d}{dx} \left( \frac{U}{U_m} \right)^{\frac{13}{8}} + 0.136 \rho U_m^2 b \left( \frac{U_m x}{b} \right)^{\frac{13}{8}} b \left( \frac{U_m x}{b} \right) =

-0.0110 \rho b U_0^2 \left( \frac{U_m}{U_0} \right)^{\frac{13}{8}} \left( \frac{x}{b} \right)^{\frac{13}{8}} \tag{13}
\]

where, \( \text{Re}_n = U_m 2b/\nu \). Here \( U_0 \)-initial outlet velocity. Taking the derivative by \( x \) on the left and rearranging it, we acquire:

\[
\frac{d}{dx} \left( \frac{U_m}{U_0} \right)^{\frac{13}{8}} =

\frac{1}{2} \left[ \frac{d}{b} \left( \frac{U_m}{U_0} \right) ^{\frac{13}{8}} + \frac{0.4517}{\lambda \text{Re}_n^{\frac{12}{7}}} \frac{d}{b} \left( \frac{U_m}{U_0} \right) ^{\frac{13}{8}} \right]

\frac{1}{1 + \frac{0.1854}{\lambda \text{Re}_n^{\frac{12}{7}}} \left( \frac{U_m}{U_0} \right) ^{\frac{13}{8}} + \frac{0.4517}{\lambda \text{Re}_n^{\frac{12}{7}}} \left( \frac{U_m}{U_0} \right) ^{\frac{13}{8}}} \right] \tag{14}
\]

Given that in the denominator (Eq. 14) the second member one order less than unity let’s change them into numerator using the method of expansion procedure the following values \( 1/r = 1 - r + r^2 - r^3 + \ldots \), keeping the first four members of series unchanged. At zero-order approximation let’s insert to the right side of Eq. 14 the value:

\[
\frac{U_m}{U_0} = \frac{N}{\sqrt{b}} \tag{15}
\]

and integrate the equation within limits by \( x \) from the end of the initial section \( x_0 \) up to the optional distance \( x \). As a result we obtain a solution for changing maximum velocity at a first approximation:

\[
\frac{U_m}{U_0} = \left[ \frac{0.1481}{A \left( \frac{b}{x_0} \right)^{0.09}} + \frac{0.01372}{A^2 \left( \frac{b}{x_0} \right)^{0.13}} \right] \exp \left[ \frac{-0.00288 x}{A^3 \left( \frac{b}{x_0} \right)^{0.27}} \right] \tag{15}
\]

where, \( \lambda = \lambda \text{Re}_n^{\frac{12}{7}} \), \( \lambda = 2h/2b, \text{Re}_n = U_m 2b/\nu, x_0 \)-polar distance. Calculations made according this equation show that towards the end of the 1st section velocity decrement correction is up to 35%.
However, length of the 1st section as the value \( \lambda \) grows is abruptly increasing (Fig. 2) and with \( \lambda > 10 \) at the distances up to \( x/b > 200 \) effect of resistance will not exceed 10%.

Comparison of the calculation results by the Eq. 15 with experimental data (Isataev et al., 2015) is shown in Fig. 3 at \( \lambda = 3 \) and \( U_0 = 4.3 \) and 63.8 m/sec.

Here, with it should be appreciated that in Fig. 3 the value of measured maximum velocity corresponds to the jet axial line. Values of maximum velocity averaged by the axis \( z \) along the overall jet height are computed in theoretical calculations. Therefore, the experimental values of maximum velocity shall be somewhat higher than the theoretically computed values.

Figure 4 shows values of turbulence level along the jet axis referred to maximum velocity for \( \lambda = 3 \). As you can see for all values of velocity \( U_0 \) from 4.3-30 m/sec turbulence levels vary the same way as for a jet at \( \lambda > 3 \).

In the 2nd section of the jet boundary layers at the wall reach the middle of the current and liquid will flow like liquid flowing in a flat channel with width \( 2b \). In this case velocity profiles shall be described by the Eq. 8 and the jet impulse equals:

\[
K = 4 \int_0^h \int_0^1 \rho U^2 \delta \eta \, dy \, dz = 4 \rho U_{_m}^2 \delta h \int_0^1 \int_0^1 \left( \frac{y}{U_{_m}} \right) \left( \frac{z}{h} \right) \, dy \, dz = 4 \rho U_{_m}^2 \delta h \int_0^1 \left( \frac{Z}{h} \right) \left( \frac{1}{6} \eta^4 + 8 \eta^3 - 3 \eta^2 \right) \, d\eta = \frac{8}{9} \rho U_{_m}^2 \delta h
\]

\[
(16)
\]

Resistance force of end walls in the section with dimension \( 4h \, dx \) shall be also defined by the Eq. 12. Inserting them into Eq. 5, we have:

\[
\frac{d}{dx} \left( \frac{8}{9} \rho U_{_m}^2 \delta h \right) = -4.15 \tau_w \delta
\]

\[
(17)
\]

Inserting the values:

\[
\delta = 0.092x, \delta = 0.238x, \tau_w = \frac{0.3164}{\left( \frac{U_{_m} b}{h} \right)^{1/4}} \rho U_{_m}^2
\]

\[
0.01963 \left( \frac{U_{_m} b}{h} \right)^{1/4} \rho U_{_m}^2
\]

into Eq. 17, after rearrangement, we obtain:

\[
\frac{\rho U_{_m}^2}{v}
\]
\[ \frac{U_m}{U_0} \left( \frac{U_m}{U_0} \right) = \frac{1}{2} \left( \frac{h}{b} \right)^{0.25} \left( \frac{U_m}{U_0} \right)^{1.25} \left( \frac{x}{b} \right) \]

At zero-order approximation assuming:

\[ \frac{U_m}{U_0} = \frac{N}{b} \left( \frac{x}{h} \right) \]

and inserting it to the right part Eq. 18, we obtain a solution at a first approximation. Integrating by \( x \) shall be carried out from the end of the first section and so on. Here with the velocity value \( U_m/U_0 \) shall be calculated by the Eq. 15 with value \( x = x_1 \). Then after integrating, we have:

\[ \frac{U_m}{U_0} = \frac{U_{m1}}{U_0} \left[ \frac{x}{b} \right] \left[ \frac{x_{1} + x_0}{b} \right] \exp \left\{ -\frac{0.01575}{h} \left( \frac{x}{b} \right)^{1.25} \left( \frac{x_{1}}{b} \right)^{1.25} \left( \frac{Re}{h} \right)^{0.25} \right\} \]

This equation shall be used at values of parameter \( \lambda \leq 3 \), since, for values \( \lambda > 3 \) the value \( x_{1}/b \leq 10 \) and reaches at \( \lambda = 25 \) and \( U_0 = 30 \text{ m/sec} \) up to \( x_{1}/b = 850 \) which is beyond the measuring range limits and area of applying jet streams.

**CONCLUSION**

Jet flow scheme between the restrictive end plates has been built. In a \( xy \) plane the jet is distributed as a free jet. In a \( xz \) plane in first section formed boundary layer on the end walls similar to the boundary layer in a uniform flow around the plates in the second section when the boundary layers are closed on the jet axis flow is similar for the flow in a flat channel. This research shows results of calculation of friction resistance of end plates affecting tendencies of free flat jet behavior. Estimate of friction at turbulent boundary layer is fulfilled. Calculation formula for the first section which describes variation of maximum jet velocity at a first approximation is acquired. These calculation data are compared with the experimental data. Also a theoretical calculation of the second section was conducted. For this study calculation formula about the maximum velocity variation was received.

**REFERENCES**


