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MICROWAVE ELECTROMAGNETICS

THE SMITH-PURCELL EFFECT. ANOMALOUSLY HIGH LEVEL OF OUTGOING WAVE EXCITATION

***S.S. Sautbekov*[1], *K.Yu. Sirenko*[2,♦], *Yu.K. Sirenko*[2], *A.Ye. Poyedinchuk*[2], *N.P. Yashina*[2], *& A.P. Yevdokymov*[2]**

[1]*Al-Farabi Kazakh National University, 71, al-Farabi Av., Almaty 050040, Republic of Kazakhstan*

[2]*O.Ya. Usikov Institute for Radio Physics and Electronics, National Academy of Sciences of Ukraine, 12 Academician Proskura St., Kharkiv 61085, Ukraine*

♦Address all correspondence to: Yu.K. Sirenko, E-mail: yks@ire.kharkov.ua

A new approach to the analysis of the Smith-Purcell effect and its wave analogs has been proposed and implemented. It is based on the rigorous models of the exact absorbing conditions method and allows obtaining sufficiently reliable numerical data on the investigated processes in the conditions of a possible resonance waves scattering. Numerical experiments based on these models relieved and investigated in detail regimes with an anomalously high level of excitation of the outgoing waves that are of great interest both for theory and applications.

KEY WORDS: *method of exact absorbing conditions, periodic grating, Smith-Purcell effect, dielectric waveguide, excitation efficiency of outgoing waves*

1. INTRODUCTION

A plane density-modulated electron beam moving at a constant velocity above an infinite one-dimensional periodic grating radiates into the environmental space homogeneous plane electromagnetic waves, which number, wavelength and direction of propagation are determined by the velocity and modulation period of the beam, and also by the length of the period of grating. Simulation of this effect (the Smith-Purcell effect [1,2]) and its wave analogs within the approximation, usually called the approximation of a given current, analysis of its various features in the spatial-time and spatial-frequency transformations of electromagnetic fields arising during their implementation – that are the main topics of this work. The wave analogues of the Smith-Purcell phenomenon we call the effects of surface-spatial mode conversion –

the surface wave of a plane dielectric waveguide whose field is in many respects similar to the field of a charged particle beam, sweeping by an exponentially decreasing part the grating surface, generates in its radiation zones bulky waves, propagating without decaying (in the lossless medium) infinitely far.

In the model based on the approximation of a given current (the approximation of a given field, in the case of the wave analogs of the Smith-Purcell effect), it is assumed that all the parameters of the electron beam remain unchanged along all the length of infinite space of its interaction with the periodic structure. Of course, this is not quite what happens in reality. But this approximation helps greatly simplify the analysis and unambiguously determine such important parameters of the simulated processes as the amount and direction of the outgoing bulk waves generated by the electron beam or by the surface wave; the energy that is allocated to these waves due a specific grating at a particular frequency [2–9]. The corresponding results, supplemented and verified by the experiments, served as basis for the creation of several fundamentally new devices for microwave technology: stable mm range coherent sources of electromagnetic oscillations operating on the Smith-Purcell effect [2,4,10]; diffraction radiation antennas with unique characteristics [11–13], etc.

Returning to the topic, seeming almost exhausted, we propose a new approach to analyzing the Smith-Purcell effect and its wave analogs resulting in a number of physical results that have never been noted before. Our approach is based on the models of the method of exact absorbing conditions [14,15]. The EAC method solves correctly and efficiently the problem of limiting the computational domain of electromagnetic open initial boundary problems [16]. Such procedures enables the carrying out robust numerical analysis under conditions of possible resonant wave scattering [15,17,18]. The new physical results are mainly related to the anomalously high level of the transformation of an inhomogeneous plane wave into one of the propagating spatial harmonics of the periodic structure when a reflecting grating is installed in field of this inhomogeneous plane wave.

We use SI, the International System of Units, for all physical parameters except the 'time' t that is the product of the natural time and the velocity of light in vacuum, thus t is measured in meters. In this paper, dimensions are omitted. According to SI, all geometrical parameters (a, b, c , etc.) are given in meters. However, this is obviously not a serious obstacle to extend the results to any other geometrically similar structure.

2. PROBLEMS OF ELECTROMAGNETIC THEORY OF GRATINGS

Correctly formulated initial boundary value problem, having solution $U\left(g,t\right)$ easily convertible into conventional amplitude-frequency characteristics of periodic in the y direction and homogeneous in the x direction reflecting grating (Fig. 1(a)), has the form [14,15]:

$$\begin{cases} \left[-\varepsilon(g)\partial_t^2 - \sigma(g)\eta_0\partial_t + \partial_y^2 + \partial_z^2\right]U(g,t) = 0; \quad g \in \Omega_{\text{int}}, \quad t > 0 \\ U(g,0) = 0, \quad \partial_t U(g,t)\big|_{t=0} = 0; \quad g = \{y,z\} \in \bar{\Omega}_{\text{int}} \\ \vec{E}_{tg}(q,t) \quad \text{and} \quad \vec{H}_{tg}(q,t) \quad \text{are continuous when crossing} \quad \Sigma^{\varepsilon,\sigma}, \\ \quad \vec{E}_{tg}(q,t)\big|_{q=\{x,y,z\}\in\Sigma} = 0, \quad U\{\partial_y U\}(l,z,t) = \mathrm{e}^{2\pi i\Phi}U\{\partial_y U\}(0,z,t) \\ \quad \text{for} \quad -h-c<z<0, \quad \text{and} \quad D\left[U(g,t) - U_p^i(g,t)\right]\big|_{g\in\mathrm{L}} = 0; \quad t \ge 0, \end{cases} \tag{1,a}$$

$$\begin{aligned} U(g,t) - U_p^i(g,t) = -\sum_{n=-\infty}^{\infty}\Bigg\{\int_0^{t-z} J_0\left[\Phi_n\left((t-\tau)^2 - z^2\right)\right]\times \\ \times\left[\int_0^l \partial_{\tilde{z}}\left[U(\tilde{g},\tau) - U_p^i(\tilde{g},\tau)\right]\Big|_{\tilde{z}=0}\mu_n^*(\tilde{y})\,d\tilde{y}\right]d\tau\Bigg\}\mu_n(y); \quad g \in \bar{\Omega}_{\text{ext}}, \quad t \ge 0. \end{aligned} \tag{1,b}$$

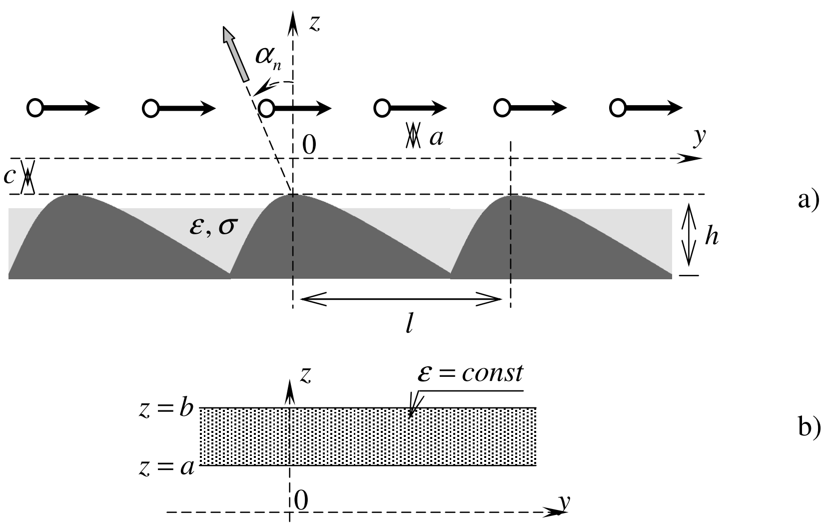

FIG. 1: Geometry of model problems: reflective grating (a) and a planar dielectric waveguide (b); $\partial_x \equiv 0$

Here, $U(g,t) = E_x(g,t)$ in the case of E-polarization and $U(g,t) = H_x(g,t)$ in the case of H-polarization; $\vec{E}(g,t) = \{E_x, E_y, E_z\}$ and $\vec{H}(g,t) = \{H_x, H_y, H_z\}$ are the electric and magnetic field vectors; $\{x,y,z\}$ are the Cartesian coordinates and $\tilde{g} = \{\tilde{y},\tilde{z}\}$; the piecewise-constant functions $\sigma(g) \ge 0$ and $\varepsilon(g) \ge 1$ are the specific conductivity and the relative permittivity of dielectric elements; $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$ is the impedance of free space; ε_0 and μ_0 are the electric and magnetic vacuum constants. The surfaces $\Sigma = \Sigma_x \times \left[|x| \le \infty \right]$ of perfectly conducting elements of a grating and the surfaces $\Sigma^{\varepsilon,\sigma}$ of discontinuities of its material parameters are assumed to be sufficiently smooth. The analysis domain Ω_{int} is the part of the Floquet channel $\text{R}=\left\{ g: 0 < y < l \right\}$ bounded by Σ_x and by the virtual boundary $\text{L} = \left\{ g \in \text{R}: z = 0 \right\}$, $\Omega_{\text{ext}} = \left\{ g \in \text{R}: z > 0 \right\}$. $J_m(...)$ are the Bessel cylindrical functions and the asterisk '$*$' stands for the complex conjugation. The transverse functions $\mu_n(y) = l^{-1/2}\exp(i\Phi_n y)$, $n = 0, \pm 1, \pm 2, \ldots$, $\Phi_n = (n + \Phi)2\pi/l$ constitute a complete orthonormal system in the cross section of Floquet channel R. Thus, for $g \in \bar{\Omega}_{\text{ext}}$ and $t > 0$ the following representations for the sought-for field are correct:

$$\begin{aligned} U(g,t) &= U_p^i(g,t) + U^s(g,t) = v_p(z,t)\mu_p(y) + \sum_{n=-\infty}^{\infty} u_{np}(z,t)\mu_n(y) \\ \text{and} \quad u_n(z,t) &= \int_0^l U^s(g,t)\mu_n^*(y)\,dy. \end{aligned} \tag{2}$$

The function $U_p^i(g,t)$ determining the pulsed wave $\left\{ \vec{E}^i(g,t), \vec{H}^i(g,t) \right\}$ coming on the boundary L from the domain Ω_{ext} is assumed to be given along with the functions $\varepsilon(g)$, $\sigma(g)$, the contours Σ_x, $\Sigma_x^{\varepsilon,\sigma}$, and the values of c and $\Phi: \text{Im}\,\Phi = 0,\ |\Phi| \le 0.5$.

The exact absorbing condition $D\left[U(g,t) - U_p^i(g,t) \right]\Big|_{g\in\text{L}} = 0$ converting the fundamentally open problem of the electrodynamic theory of gratings into a closed one and allowing the use for it solving the standard calculating schemes of the finite difference method [16] or the finite element method [19] does not distort the physics of processes modeled by mathematical means. This is also valid for the resonance processes appearing due to excitation of weakly decaying free oscillations of the electromagnetic field in the grating. The analytical form of this condition is obtained by putting the observation point g in the representation (1,b) onto the virtual boundary L.

The solution $\tilde{U}(g,k)$ to the problem

$$\begin{cases} \left[\partial_y^2 + \partial_z^2 + \tilde{\varepsilon}(g)k^2 \right]\tilde{U}(g,k) = 0; \quad g \in \Omega_{\text{int}} \\ \tilde{\vec{E}}_{tg}(q,k), \quad \tilde{\vec{H}}_{tg}(q,k) \quad \text{are continuous when crossing} \quad \Sigma^{\varepsilon,\sigma} \\ \quad \text{and boundary} \quad \text{L} \times \left[|x| \le \infty \right], \quad \tilde{\vec{E}}_{tg}(q,k)\Big|_{q=\{x,y,z\}\in\Sigma} = 0, \quad \text{and} \\ \quad \tilde{U}\left\{ \partial_y \tilde{U} \right\}(l,z,k) = e^{2\pi i\Phi}\tilde{U}\left\{ \partial_y \tilde{U} \right\}(0,z,k) \quad \text{for} \quad -h-c \le z \le 0, \end{cases} \tag{3,a}$$

$$\tilde{U}(g,k)=\tilde{U}_p^i(g,k)+\tilde{U}^s(g,k)=A_p(k)\mathrm{e}^{-i\Gamma_p z}+\sum_{n=-\infty}^{\infty}B_{np}(k)\mathrm{e}^{i\Gamma_n z}\mu_n(y);\quad g\in\bar{\Omega}_{\text{ext}}\quad(3,\text{b})$$

and the solution $U(g,t)$ to the problem (1) can be related [15] by the following integral transform

$$\tilde{f}(k)=\int_0^{\infty}f(t)\mathrm{e}^{ikt}\,dt\;\leftrightarrow\;f(t)=\frac{1}{2\pi}\int_{i\alpha-\infty}^{i\alpha+\infty}\tilde{f}(k)\mathrm{e}^{-ikt}\,dk;\quad 0\le\alpha\le\operatorname{Im}k. \qquad (4)$$

Here, k is a complex wavenumber (frequency parameter or frequency), $\tilde{\varepsilon}(g)=\varepsilon(g)+i\eta_0\sigma(g)/k$, $\Gamma_n=\sqrt{k^2-\Phi_n^2}$ ($\operatorname{Re}\Gamma_n\operatorname{Re}k\ge 0$, $\operatorname{Im}\Gamma_n\ge 0$) and $\Phi_n=(n+\Phi)2\pi/l$ are the vertical and horizontal propagation constants for spatial harmonics (plane waves) propagating in the domain Ω_{ext} with attenuation (when $\operatorname{Im}\Gamma_n>0$) or without it (when $\operatorname{Im}\Gamma_n=0$). According to (4), the time-dependence for monochromatic components of any signal is $\exp(-ikt)$. If we correlate (1,b) with (3,b), it becomes evident that $A_n(k)\leftrightarrow v_n(0,t)$ and $B_{np}(k)\leftrightarrow u_{np}(0,t)$.

Consider now frequencies k such that $\operatorname{Re}k>0$ and $\operatorname{Im}k=0$ (physical values of the frequency parameter $k=2\pi/\lambda$, λ is the wavelength). In the frequency domain, a periodic structure is characterized by the reflection coefficients $R_{np}(k)$ given by the formula $R_{np}(k)=B_{np}/A_p=\tilde{u}_{np}(0,k)/\tilde{v}_p(0,k)$. The elements $R_{np}(k)$ of the generalized scattering matrix $\left\{R_{np}(k)\right\}_{n,p=-\infty}^{\infty}$ are related by the energy balance equations

$$\sum_{n=-\infty}^{\infty}\left|R_{np}\right|^2\begin{Bmatrix}\operatorname{Re}\Gamma_n\\ \operatorname{Im}\Gamma_n\end{Bmatrix}=\begin{Bmatrix}\operatorname{Re}\Gamma_p+2\operatorname{Im}R_{pp}\operatorname{Im}\Gamma_p\\ \operatorname{Im}\Gamma_p-2\operatorname{Im}R_{pp}\operatorname{Re}\Gamma_p\end{Bmatrix}-\frac{k^2}{\beta_0}\begin{Bmatrix}W_1\\ W_2\end{Bmatrix};\quad p=0,\pm1,\pm2,... \qquad (5)$$

and by the reciprocity relations

$$R_{np}(\Phi)\Gamma_{-n}(-\Phi)=R_{-p,-n}(-\Phi)\Gamma_p(\Phi);\quad n,p=0,\pm1,\pm2,..., \qquad (6)$$

which are the corollaries from the Pointing theorem on complex power and the Lorentz lemma [15]. In (5), we have used the following denotations:

$$\beta_0=\begin{Bmatrix}\varepsilon_0\\ \mu_0\end{Bmatrix},\quad W_1=\frac{\eta_0\varepsilon_0}{k}\int_{\Omega_{\text{int}}}\sigma(g)\left|\tilde{\tilde{E}}(g,k)\right|^2dg,\quad\text{and}$$

$$W_2=\begin{Bmatrix}+\\ -\end{Bmatrix}\int_{\Omega_{\text{int}}}\left[\mu_0\left|\tilde{\tilde{H}}(g,k)\right|^2-\varepsilon(g)\varepsilon_0\left|\tilde{\tilde{E}}(g,k)\right|^2\right]dg;\quad\begin{Bmatrix}E-\text{case}\\ H-\text{case}\end{Bmatrix}.$$

Every harmonic of the field $\tilde{U}^s(g,k)$, for which $\operatorname{Im}\Gamma_n = 0$ and $\operatorname{Re}\Gamma_n > 0$, is a homogeneous plane wave propagating away from a grating at the angle $\alpha_n = -\arcsin(\Phi_n/k)$ which is measured anticlockwise from the z-axis in the plane $y0z$ (Fig. 1(a)). The point $k = k_n^{\text{grat}} = |\Phi_n|$ – by passing which the evanescent spatial harmonic (an inhomogeneous plane wave) turns into a propagating one – is called a threshold point.

For $\operatorname{Re}\Gamma_p > 0$, the angle $\alpha_p^i = \arcsin(\Phi_p/k)$ is the angle of incidence of the wave $\tilde{U}_p^i(g,k)$ coming onto a grating. According to (5), the values $W_{\text{abs}}(k) = (k^2 W_1)\big/(\beta_0|\Gamma_p|)$ and $W_{np}(k) = \left(|R_{np}|^2 \operatorname{Re}\Gamma_n\right)\big/|\Gamma_p|$ determine in this case the relative part of energy lost to absorption and directed by a grating into the relevant spatial harmonic.

If a grating is excited by an inhomogeneous plane wave ($\operatorname{Im}\Gamma_p > 0$), the near-field to far-field conversion efficiency is determined by the value of $\operatorname{Im}R_{pp}$ (see (5)), which in this case is nonnegative and

$$2\operatorname{Im}R_{pp} = \sum_n W_{np} + W_{\text{abs}}. \tag{7}$$

As it follows from (6) and the equalities $\Phi_n(\Phi) = -\Phi_{-n}(-\Phi)$, $\Gamma_n(\Phi) = \Gamma_{-n}(-\Phi)$, one can study the excitation of a grating by an inhomogeneous plane wave in the context of conventional for the gratings theory diffraction problems: a structure is excited by homogeneous plane wave $\tilde{U}_{-n}^i(g,k,-\Phi)$ and the coefficients of conversion into evanescent ($-p$)-th spatial harmonics $R_{-p,-n}(-\Phi)$ are calculated.

3. SIMULATION OF THE SMITH-PURCELL PHENOMENON AND ITS WAVE ANALOGS

The field of a density-modulated electron beam, whose instantaneous charge density can be written as $\rho\delta(z-a)\exp\left[i\left((k/\beta)y - kt\right)\right]$, $a \geq 0$ is a H-polarized field with

$$\tilde{H}_x(y,z,k) = 2\pi\rho\beta\exp\left\{i\left[\sqrt{k^2-(k/\beta)^2}\,|z-a| + (k/\beta)\,y\right]\right\}\left[|z-a|/(z-a)\right]; \quad z \neq a \tag{8}$$

[2,3]. Here, $\delta(...)$ is the Dirac delta-function, ρ and k are the modulation amplitude and the modulation frequency of the beam, and $\beta < 1$ is its relative velocity.

From (3,b) and (8) it follows that the beam-generated field (scattered field $\tilde{U}^{s}(g,k)$ of the grating, placed into the proper field of the electron beam) can be obtained from the solution $\tilde{U}(g,k)=\tilde{U}_{p}^{i}(g,k)+\tilde{U}^{s}(g,k)$ of the problem (3) for H -case and for incident wave $\tilde{U}_{p}^{i}(g,k)=A_{p}(k)\exp(-i\Gamma_{p}z)\mu_{p}(y),\ \ 0\le z<a$ with $A_{p}(k)=-2\pi\rho\beta\sqrt{l}\exp\left[-ka\sqrt{(1/\beta)^{2}-1}\right]$ and $\Phi_{p}=k/\beta$ ($\operatorname{Re}\Gamma_{p}=0,\ \operatorname{Im}\Gamma_{p}>0$).

Represent now the x-component of E - or H -polarized eigenwave of a planar waveguide (Fig. 1(b)) as

$$\tilde{U}(g,\bar{\chi})=\begin{cases}\bar{A}\exp\left[i\Gamma(\bar{\chi})(z-b)+i\bar{\chi}y\right]; & z\ge b\\ \left[C\exp\left[-i\Gamma_{\varepsilon}(\bar{\chi})(z-b)\right]+D\exp\left[i\Gamma_{\varepsilon}(\bar{\chi})(z-a)\right]\right]\times & \\ \quad\times\exp(i\bar{\chi}y); & 0\le a\le z\le b\\ \underline{A}\exp\left[-i\Gamma(\bar{\chi})(z-a)+i\bar{\chi}y\right]; & z\le a.\end{cases} \tag{9}$$

Here, $\bar{\chi}$ is the complex-valued longitudinal propagation number for the eigenwave $\tilde{U}(g,k,\bar{\chi})$; $\Gamma(\chi)=\sqrt{k^{2}-\chi^{2}}$ (a branch of the square root is determined by the point χ location on the two-sheeted Riemann surface X with the algebraic branch points $\chi^{\pm}=\pm k$ [13]); $\Gamma_{\varepsilon}(\chi)=\sqrt{k^{2}\varepsilon-\chi^{2}}$ (one can choose any branch of the root). On the axis $\operatorname{Re}\chi$ of the first (physical) sheet of X we have $\operatorname{Re}\gamma(\chi)\ge 0$ and $\operatorname{Im}\gamma(\chi)\ge 0$. From the physically evident requirement it follows that the field $\tilde{U}(g,k,\bar{\chi})$ does not contain waves incoming from infinity.

An eigenwave, having field strength exponentially decreasing when moving from the dielectric layer ($\operatorname{Im}\gamma(\bar{\chi})>0$), is a surface wave, otherwise ($\operatorname{Im}\gamma(\bar{\chi})\le 0$) it is a leaky wave. A wave $\tilde{U}(g,k,\bar{\chi})$ is called a real wave if $\operatorname{Im}\chi=0$, and a complex wave if $\operatorname{Im}\chi\ne 0$. A real surface wave is a true wave. Its relative phase velocity $\beta=k/\bar{\chi}<1$ and thus it is a slow wave.

If a real, E - or H -polarized surface wave of plane dielectric waveguide covers by its exponentially decreasing component the surface $z=-c$ of periodic structure, then the field $\tilde{U}^{s}(g,k)$ generated as a result of such interaction, is defined by the solution $\tilde{U}(g,k)=\tilde{U}_{p}^{i}(g,k)+\tilde{U}^{s}(g,k)$ of the problem (3) for E - or H -case and for incident wave $\tilde{U}_{p}^{i}(g,k)=A_{p}(k)\exp(-i\Gamma_{p}z)\mu_{p}(y),\ \ 0\le z<a$ with $A_{p}(k)=\underline{A}(k)\sqrt{l}\exp(i\Gamma_{p}a)$ and $\Phi_{p}=\bar{\chi}$ ($\operatorname{Re}\Gamma_{p}=0,\ \operatorname{Im}\Gamma_{p}>0$). It is easy to arrive to such conclusion comparing representations (3,b) and (9).

Thus, the analysis of Smith-Purcell effect and its wave analogs is reduced to the solution of problem (3) that is to the calculation of the secondary field $\tilde{U}^s(g,k)$ of the grating placed in the field of a plane wave $\tilde{U}_p^i(g,k)$. And, in general, it does not matter what kind of wave it is – homogeneous, propagating without decaying, or inhomogeneous, with field exponentially decreasing during the propagation. The first case is easily recalculated to the second one using simple formulas analytically representing the so-called reciprocity relations (6).

4. THE SMITH-PURCELL EFFECT. ANOMALOUSLY HIGH LEVEL OF EXCITATION OF OUTGOING WAVES

Let us put the period l of all the grating under consideration equal 2π – this will make it easy to treat all the analytic and numerical results presented in the paper in terms of those dimensionless parameters that are conventionally used in the theory of periodic structures. These are coordinates $\{Y,Z\}=\{2\pi y/l, 2\pi z/l\}$, time $\tau = 2\pi t/l$, frequency $\kappa = l/\lambda = kl/2\pi$.

We excite a lamellar grating (see lower fragment in Fig. 2) by a pulsed H -polarized wave

$$\begin{gathered} U_1^i(g,t): v_1(0,t) = 4\sin\left[\Delta k\left(t-\tilde{T}\right)\right]\left(t-\tilde{T}\right)^{-1}\cos\left[\tilde{k}\left(t-\tilde{T}\right)\right]\chi\left(\bar{T}-t\right) = F_1(t); \\ \Phi = 0.01, \quad \tilde{k} = 0.5, \quad \Delta k = 0.45, \quad \tilde{T} = 150, \quad \bar{T} = 300. \end{gathered} \tag{10}$$

Here, $\tilde{k}$, Δk, $\tilde{T}$, and $\bar{T}$ stand for the central frequency of the signal, its band ($\tilde{k}-\Delta k \le k \le \tilde{k}+\Delta k$), delay time, and duration, respectively; $\chi(...)$ is the Heaviside step function. A detailed description of the temporal and spectral characteristics of the pulse $F_1(t)$ could be found in the book [15]. In the frequency band $0.05 \le k \le 0.95$ occupied by the pulse (10), only one of the spatial harmonics of the secondary field $\tilde{U}^s(g,k)$ of grating propagates without decaying: $k_0^{\text{grat}} = 0.01$, $k_{-1}^{\text{grat}} = 0.99$ and $k_1^{\text{grat}} = 1.01$. We identify the field of the wave $\tilde{U}_1^i(g,k)$ with the proper field of the electron beam moving above the grating with velocity $\beta = k/\Phi_1$, $\Phi_1 = 1.01$. The efficiency of its transformation into the field of the outgoing wave is determined by $W_{01}(k) = 2\,\text{Im}\,R_{11}(k)$ (see formula (7)). For values θ equal to 0.1, 0.3, and 0.5, and values $\delta = 0.1 \div 0.9$ the efficiency varies within the limits $0 \le W_{01}(k) < 9.0$ (Fig. 2).These are the usual rates for the situation in question [2,5,8]: a low level of energy extraction into the zero spatial harmonic over the greater part of the considered frequency range and local spikes of $W_{01}(k)$ due to low-Q resonances on TEM -wave propagating in the grating grooves.

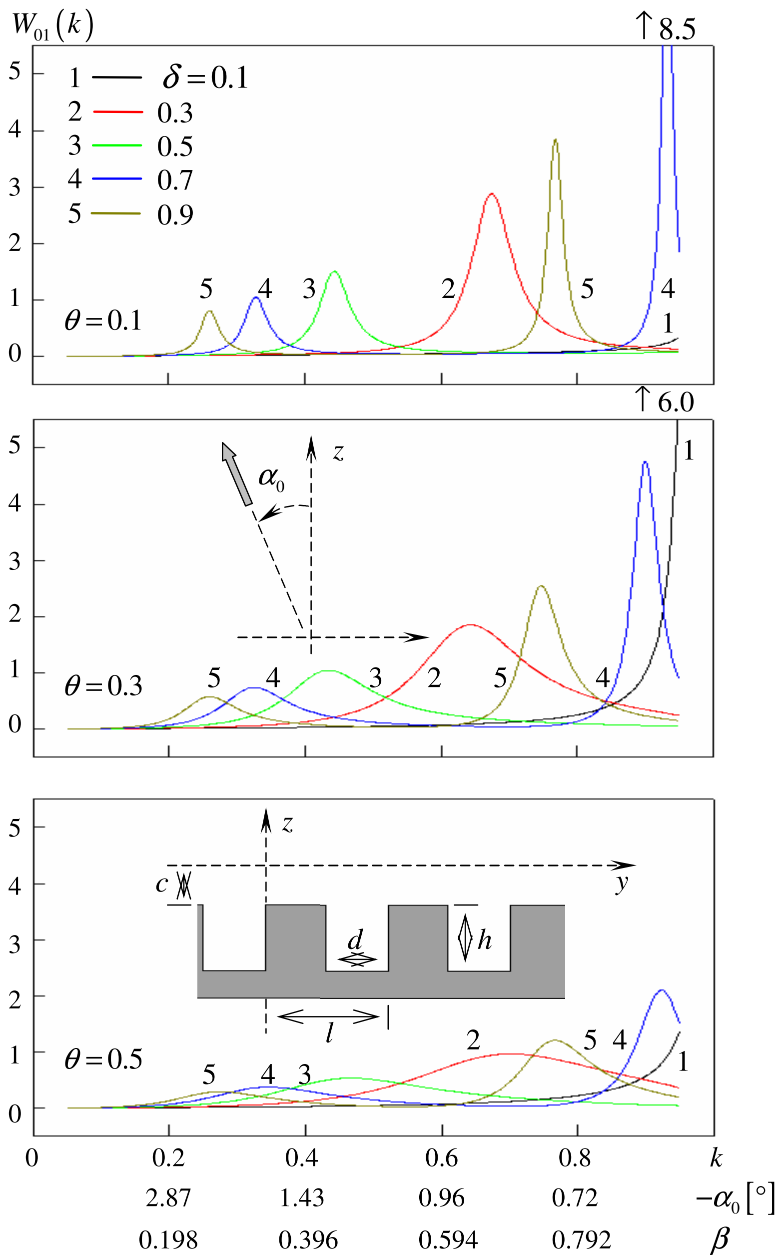

FIG. 2: H -polarization. The efficiency of radiation at zero spatial harmonic at different values of the geometrical parameters of the reflective grating: $p = 1$; $\Phi = 0.01$; $c = 0.02l$; $h = \delta l$; $d = \theta l$.

The situation is dramatically changed by the appearing in the grating's grooves at θ equal to 0.7 and 0.9, of propagating TM_{01} -wave ($k_1^{\text{wave}} \approx 0.714$ for $\theta = 0.7$ and

$k_1^{\text{wave}} \approx 0.555$ for $\theta = 0.9$; $k_m^{\text{wave}} = n\pi/d$, $m = 1,2,...$ are the cut-off points for TM_{0m}- and TE_{0m}-waves in an empty parallel-plane waveguide of width d). The values of the function $W_{01}(k)$ in the neighborhood of the points $k = \operatorname{Re}\bar{k}$ corresponding to the resonances on the trapped in the grooves TM_{01}-wave reach dozens and hundreds. The quality factor $Q = \operatorname{Re}\bar{k}/2\left|\operatorname{Im}\bar{k}\right|$ of these resonances and eigen field patterns at complex eigen frequencies $\bar{k} = \operatorname{Re}\bar{k} + i\operatorname{Im}\bar{k}$, we find out by exciting the grating with the narrow band Gaussian pulses

$$\begin{gathered} U_1^i(g,t): v_1(0,t) = \exp\left[-\left(t-\tilde{T}\right)^2/4\tilde{\alpha}^2\right]\cos\left[\tilde{k}\left(t-\tilde{T}\right)\right]\chi\left(\bar{T}-t\right) = F_2(t); \\ \Phi = 0.01, \quad \tilde{k} = \operatorname{Re}\bar{k}, \quad \tilde{\alpha} = 40, \quad \tilde{T} = 200, \quad \bar{T} = 400 \end{gathered} \tag{11}$$

and watching the $U(g,t)$-field, $g \in \Omega_{\text{int}}$ damping process for $t > \bar{T}$ (free oscillations regime) [20–23].

The result of one of such computational experiments is presented in Fig. 3. The quality of the free oscillation excited in the grating, as far as can be estimated from the right fragment of Fig. 3 is rather high. Comparison of the field amplitudes $U(g_1,t)$ at time instants $t \approx 400$ and $t \approx 1000$ (determining the actual rate of decrease of the signal envelope $A\exp\left(\operatorname{Im}\bar{k}\cdot t\right)$, $t > 400$) allows us to estimate it by value of $Q \approx 12000$.

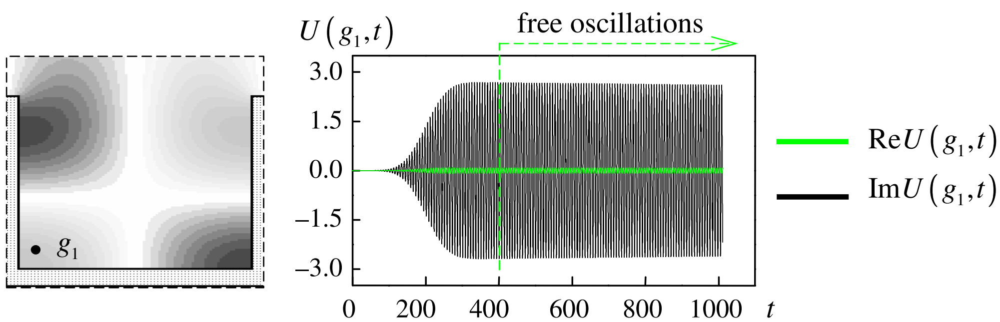

FIG. 3: Excitation of the grating by a narrow-band H-polarized pulse (11): $\tilde{k} = \operatorname{Re}\bar{k} = 0.884$, $c = 0.2l$; $\theta = 0.9$; $\delta = 0.9$. Pattern $U(g,t) = H_x(g,t)$, $g \in \Omega_{\text{int}}$, $t = 600$ and function $U(g,t)$ in the antinode of the free oscillation field.

What is the reason of the excitation such high-Q field oscillation, providing the anomalously high level of energy transformation into the zero harmonic of the secondary field $\tilde{U}^s(g,k)$ of the grating? To answer this question, we recall first of all that the most general characteristic adequately describing the principal features of the processes of resonant scattering of plane waves by any transparent or reflective waveguide-type grating [15,24–26] can be represented by a couples of numbers $\{N,M\}$. Here $N=\sum_n \operatorname{Re}\Gamma_n/|\Gamma_n|$ – is the number of spatial harmonics propagating without decaying in the reflection and transmission zones of the grating, and M is the total number of propagating modes with mismatched propagation constants in the regular intervals of all waveguide channels connecting these zones. The couple $\{N,M\}=\{1,2\}$ ($M=\sum_m \operatorname{Re}\gamma_m/|\gamma_m|$, $\gamma_m=\sqrt{k^2-(m\pi/d)^2}$, $\operatorname{Re}\gamma_m,\ \operatorname{Im}\gamma_m\geq 0$, $m=0,1,2,...$) corresponds to the situation of interest: in the reflection zone of the grating, only zero spatial harmonic propagates without decaying, and TEM -, TM_{01} -waves propagate in the groove without decaying.

It was noted in [15,26] that ultrahigh-Q free field oscillations in lamellar grating operating in the regime $\{N,M\}=\{1,2\}$ can be excited as a result of mode coupling of eigen oscillations of the first and second families belonging to the same symmetry class as well as operation within the parameter range, where the slightest change of one of the parameters results in the change of regimes from $\{1,2\}$ to $\{1,1\}$. The mode coupling, caused by the closing in (coincidence) of the $\operatorname{Re}\bar{k}$ values corresponding to one of the free oscillations on the TEM -wave in the grating groove (one of the free oscillations of the first family) and to one of the free oscillations on the TM_{01} -wave (one of the free oscillations of the second family) results in a sharp, practically unlimited growth of Q of the second of them. This effect is point-like (on the plane "frequency, one of the other parameters of the problem"), and it is almost impossible to find it by chance, without a preliminary solution of spectral problems [15,26]. Obviously, this is not our case. The option with mode change remains, and just its implementations in a pointed out anomalously high spikes of function $W_{01}(k)$ are confirmed by the results shown in Figs. 4 and 5.

The maximum values $W_{01}^{\max}=W_{01}(\operatorname{Re}\bar{k})$ of the function $W_{01}(k)$ calculated in the frequency range $0.55\leq k\leq 0.85$ for the grating of parameters $\theta=0.9$, $\delta=0.3$ and with parameter Φ values approaching to 0.01 (mode $\{1,2\}$), monotonically increase (Table 1). The Q-factor of free oscillations formed by TM_{01} counter waves in the grating groove also increases (see left fragment in Fig. 5). We find it out by exciting the grating with a pulse wave $U_0^i(g,t): v_0(0,t)=F_2(t)$; $\Phi=0.01\div 0.15$, $\tilde{k}=\operatorname{Re}\bar{k}$, $\tilde{\alpha}=40$, $\tilde{T}=200$, $\bar{T}=400$, and observing how fast the function $\operatorname{Re}u_{-10}(t)$ (the near field of the periodic structure) decreases after the source is turned off at the time instants $\bar{T}\leq t\leq 1500$.

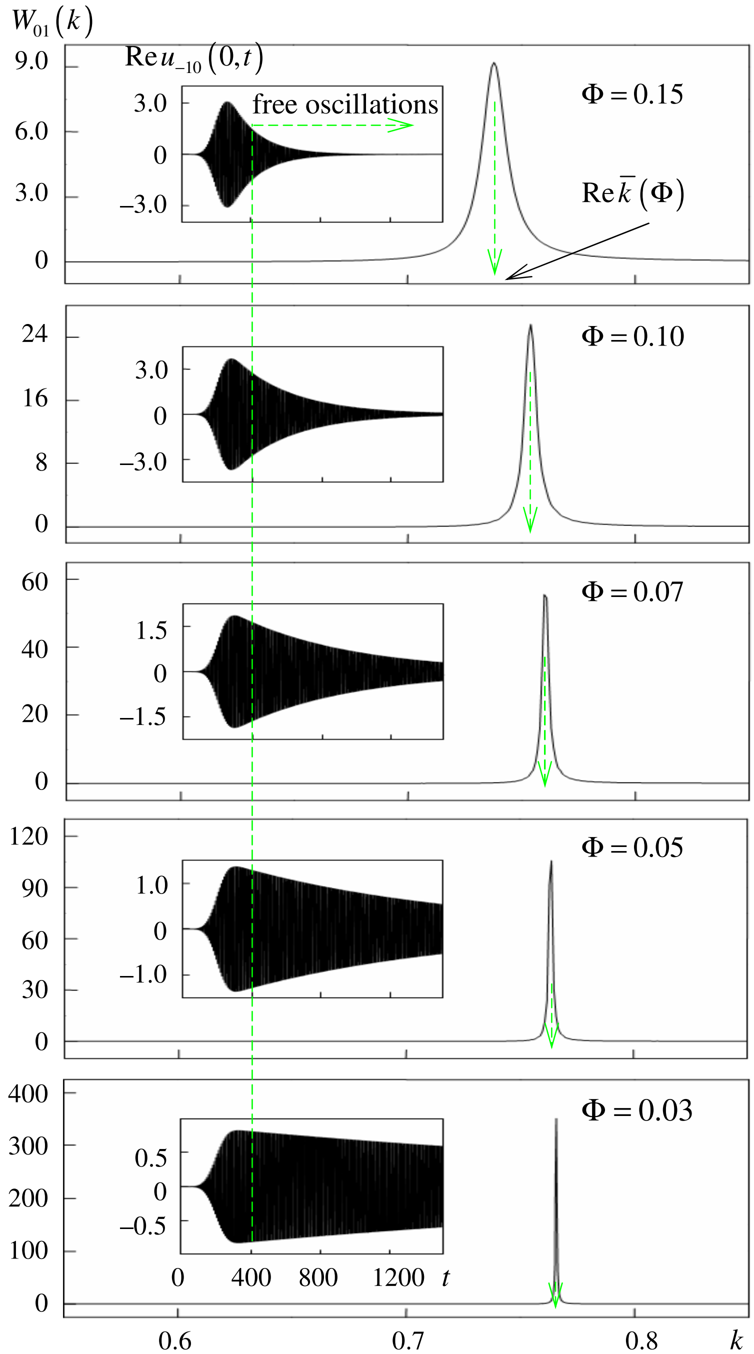

FIG. 4: H -polarization. Anomalously high radiation efficiency at zero spatial harmonic: $\theta = 0.9$; $\delta = 0.3$; $c = 0.02l$.

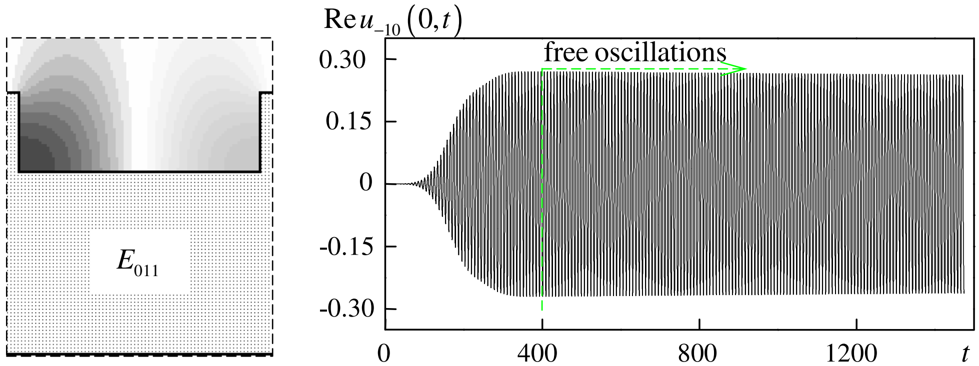

FIG. 5: H -polarization. Excitation of high-Q free oscillations of the field on the TM_{01} -wave in the grating groove: $p = 0$; $\Phi = 0.01$; $\theta = 0.9$; $\delta = 0.3$; $c = 0.02l$.

The smaller the value of Φ is, the closer we are to the "boundary" $\Phi = 0$, separating the grating operation regime $\{1,2\}$ from $\{1,1\}$, in the case of a zeroing of the value Φ, the TM_{01} -wave in the grating's groove is not excited by virtue of that the planes of grating's symmetry divide the groves in half.

TABLE 1: The efficiency of transformation

$\Phi =$	$W_{01}^{\max} = W_{01}\left(\mathrm{Re}\,\bar{k}\right) \approx$	$\bar{k} = \mathrm{Re}\,\bar{k} + i\,\mathrm{Im}\,\bar{k} \approx$	$Q = \mathrm{Re}\,\bar{k} / 2\left\vert \mathrm{Im}\,\bar{k} \right\vert \approx$
0.15	9	$0.7380 - i0.0063628$	58
0.10	26	$0.7540 - i0.0029640$	127
0.07	56	$0.7605 - i0.0012770$	298
0.05	105	$0.7635 - i0.0007841$	486
0.03	350	$0.7657 - i0.0002944$	1300
0.01	520	$0.7670 - i0.0000377$	10160

So, the reason for the abnormally high level of energy takeoff from the electron beam into the zero spatial harmonic of lamellar grating is established. Sure, a similar effect can be implemented also when the field of the H -polarized surface wave of a planar dielectric waveguide but not the field of charged particle beam, becomes a source of excitation. Thus, the case considered above: $\mathrm{Re}\,\bar{k} = 0.767$, $\Phi = 0.01$ - corresponds to the velocity $\beta = k/\Phi_1 \approx 0.759$ and the beam modulation frequency $k = 0.767$ and, in the same time, the propagation constant $\bar{\chi} = \Phi_1 = 1.01$ (the delay coefficient) and the frequency $k = 0.767$ of the surface wave.

We now demonstrate one of the possible ways of practical application of the theoretical result obtained here. While exciting a grating of parameters $\theta = 0.9$ and $\delta = 0.3$ with an ultra-long quasimonochromatic H -polarized pulse

$$U_1^i(g,t): v_1(0,t) = P(t)\cos\left[\tilde{k}\left(t-\tilde{T}\right)\right] = F_3(t);$$
$$\Phi = 0.01, \quad \tilde{k} = 0.767, \quad \tilde{T} = 0.5, \quad P(t): 0.1-5-9995-10000 \tag{12}$$

($\tilde{k}$ is the central frequency of the signal and $P(t)$: $t_1 - t_2 - t_3 - t_4$ is its trapezoidal envelope, which equals unit for $t_2 < t < t_3$ and is zero for $t < t_1$ and $t > t_4$), we see that the grating effectively accumulate input energy in a near field (see Fig. 6 – amplitudes $u_{\pm 11}(0,t)$ and Fig. 7 – $|U(g,t)|$, $g \in \Omega_{\text{int}}$, $t \approx 10000$).

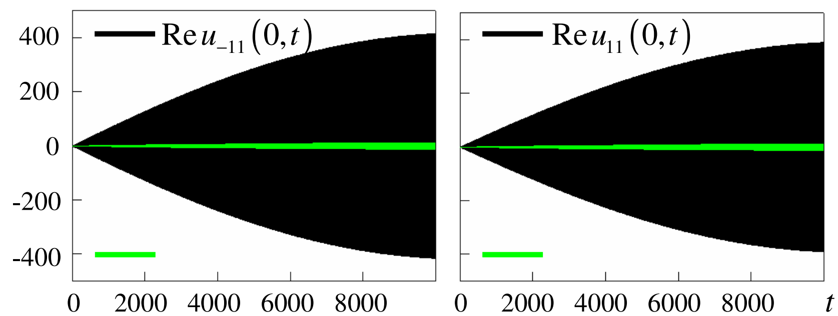

FIG. 6: H -polarization. The accumulation of energy in the near field of a grating excited by a H -polarized quasimonochromatic pulse (12): $\Phi = 0.01$; $\theta = 0.9$; $\delta = 0.3$; $c = 0.02l$. The imaginary parts of the amplitudes $u_{\pm 11}(0,t)$ and $u_{01}(0,t)$ practically do not differ from zero.

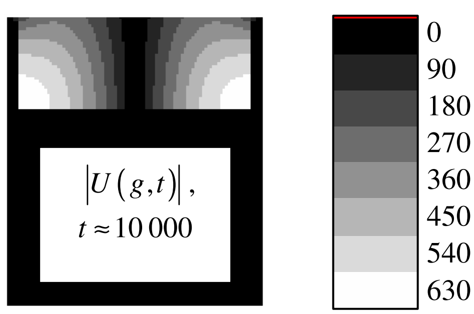

FIG. 7: The rate limits for the field strength in the grating groove at the end of the accumulation process

If at some time instant $t = 10000$ to violate somehow the conditions necessary for this (to make the frequency $k = 0.767$ non resonant), then all accumulated energy will be shot into free space in a short (with an effective duration not much greater than $2h$)

powerful pulse $U_0^s(g,t)=u_{01}(z,t)\mu_n(y)$ [27–30], having smaller in the amplitude precursor, which was radiated throughout the entire time interval $0<t\le 10000$. This effect can be used for designing a new class of diffraction antennas comprising two functions: creation and radiation of powerful short RF pulses.

5. CONFIRMATION OF THE CONCLUSION ABOUT THE REASONS OF ANOMALOUSLY HIGH LEVEL OF EXCITATION OF OUTGOING WAVES

For confirming the conclusion made in the previous section, let us consider the results of a computational experiment in which a lamellar grating with a complex structure of a period (see Fig. 8: $\varepsilon_1=1.0$, $d_1=\theta_1 l=0.48l$, $d_2=\theta_2 l=0.48l$, $h=\delta l=0.7l$) was installed in the field of H -polarized wave $U_1^i(g,t): v_1(0,t)F_1(t)$; $\Phi=0.2$, $\tilde{k}=0.5$, $\Delta k=0.25$, $\tilde{T}=150$, $\bar{T}=300$.

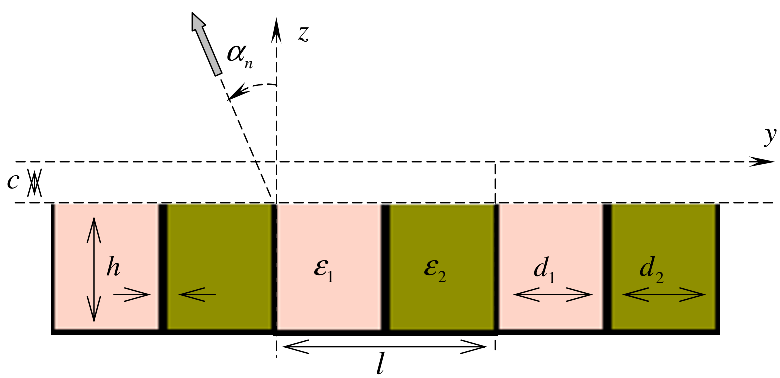

FIG. 8: The geometry of the lamellar grating with a complex structure of a period

Within the frequency band $0.25\le k\le 0.75$, occupied by pulse $U_1^i(g,t)$, the only one of the spatial harmonics of the secondary field $\tilde{U}^s(g,k)$ of grating propagates without decaying: $k_0^{\text{grat}}=0.2$, $k_{-1}^{\text{grat}}=0.8$ and $k_1^{\text{grat}}=1.2$. In the same frequency band, in the grooves of the grating with $1.0<\varepsilon_2\le 1.3$ only TEM -waves with unmatched propagation constants $\gamma_{0(1)}=k$ and $\gamma_{0(2)}=k\sqrt{\varepsilon_2}$ ($k_{1(1)}^{\text{wave}}\approx 1.04$, $0.914\le k_{1(2)}^{\text{wave}}\le 1.04$) propagate without decaying. So, again we are dealing with the grating operation regime with couple of numbers $\{1,2\}$. Reducing the value ε_2 to a value just slightly exceeding unity, we come very close to the boundary, beyond which the regime $\{1,2\}$ is replaced by the regime $\{1,1\}$; when $\varepsilon_2=1.0$, the propagation constants of TEM -waves in the grooves coincide.

We identify the field of the wave $\tilde{U}_1^i(g,k)$, as before, with the proper field of the electron beam moving above the grating with velocity $\beta = k/\Phi_1$, $\Phi_1 = 1.2$. Information on the efficiency of its transformation into the field of outgoing at an angle α_0 wave for values ε_2 equal to 1.3, 1.2, and 1.1 is presented in Fig. 9 and Table 2. Just as in the case considered in the previous section, the regular behavior of functions $W_{01}(k)$ is violated by sharp spikes in the neighborhood of points $k = \mathrm{Re}\,\bar{k}$. The Q-factor of the resonances corresponding to these spikes, and the values $W_{01}^{\max} = W_{01}(\mathrm{Re}\,\bar{k})$ increase indefinitely for $\varepsilon_2 \to 1.0$. This trend is clearly visible in the graphs of Fig. 9. The dependences $\mathrm{Re}\,u_{11}(0,t)$ for $t > \bar{T}$ presented here make it possible to estimate the rate of damping of free oscillations of the field near the grating, or, what is the same, about their Q-factor.

It can be confirmed that for any given $W > 0$ it is always possible to specify values $\varepsilon_2 > 1.0$, $\delta > 0$ and $k = \mathrm{Re}\,\bar{k}$ such that the conversion efficiency $W_{01}(\mathrm{Re}\,\bar{k})$ of an inhomogeneous plane wave $\tilde{U}_1^i(g,k)$ into a homogenous outgoing wave will exceed the value W. Only fixing the corresponding effect with each step along ε_2, approaching to the value $\varepsilon_2 = 1.0$, will be increasingly difficult. There is a high probability that using standard frequency-domain methods it will simply be missed because of insufficiently fine sampling of the varying parameter.

TABLE 2: Efficiency of transformation

$\varepsilon_2 =$	$W_{01}^{\max} = W_{01}(\mathrm{Re}\,\bar{k}) \approx$	$\mathrm{Re}\,\bar{k} \approx$
1.3	34	0.283
1.2	59	0.288
1.1	210	0.2928

6. CONCLUSIONS

Considerable amount of efficient devices in modern vacuum and optoelectronics, accelerator and microwave technology are oriented in their conceptual schemes to the implementation of the Smith-Purcell effect or its wave analogs. The theory of these effects, developed mainly using the approximation of a given current (given field), opens up a lot of possibilities for applications. But, as it turns out, a problem-oriented analysis can significantly expand the range of these possibilities. And with present work we open a series of papers devoted to such a study. For the begining, as well as here, we plan to consider the problems for which solution it is sufficient to involve approaches based on the approximation of a given current. Then we move on to more realistic models and to problems whose solution will allow us to obtain qualitatively new physical results, important for both theory and applications.

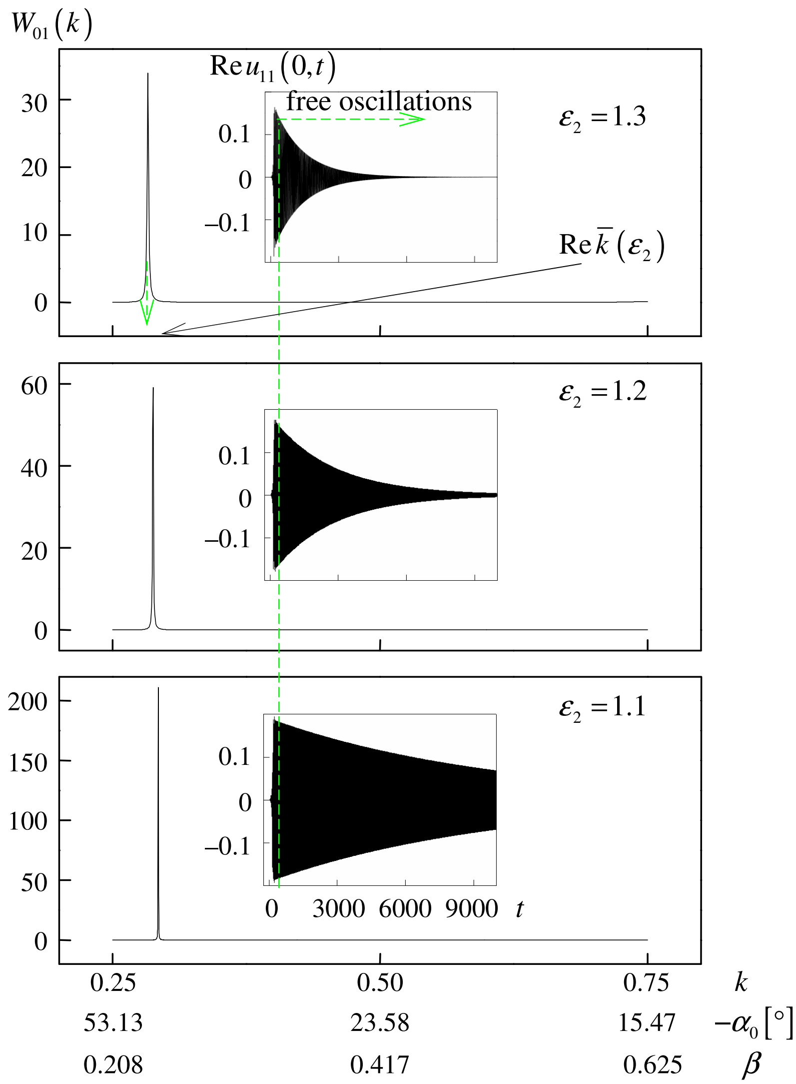

FIG. 9: H -polarization. Anomalously high radiation efficiency at zero spatial harmonic: $p = 1$; $\Phi = 0.2$; $\theta_1 = \theta_2 = 0.48$; $\varepsilon_1 = 1.0$; $\delta = 0.7$; $c = 0.02l$.

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