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About a class of n-order elliptic systems in the plane with a singular line and Fuchs operator in the differential part

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Abstract. In this article a Robin problem for a class of n-order elliptic systems with a singular line in an unbounded angular domain is solved.

Keywords. n-order, elliptic system, singular line, boundary value problem, angular domain, Wronskian, fundamental system.

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1 Introduction

Let $0 < \varphi_0 \leq 2\pi$, $G = \{z = re^{i\varphi} : 0 \leq r < \infty, 0 \leq \varphi \leq \varphi_0\}$.

We consider the equation

$$\sum_{j=1}^{n} f_j(\varphi)\left(2\bar{z}\frac{\partial}{\partial \bar{z}}\right)^j V + f_{n+1}(\varphi)V + r^{\alpha}\frac{f_{n+2}(\varphi)}{(y-k_1x)^{\alpha}}\overline{V} = r^{\nu}\frac{f_{n+3}(\varphi)}{(y-k_2x)^{\alpha}} \quad (1)$$

in G, where $f_j(\varphi) \in C[0,\varphi_0]$, $(j = 1,2,\dots,n+3)$, $f_n(\varphi) \neq 0$ for all $\varphi \in [0,\varphi_0]$, $\nu > 2+\alpha$, $k_1 = \tan\varphi_1$, $k_2 = \tan\varphi_2$, $0 < \alpha < 1$, $0 < \varphi_1, \varphi_2 < \varphi_0$,

$$\frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right), \quad \frac{\partial^k V}{\partial \bar{z}^k} = \frac{\partial}{\partial \bar{z}}\left(\frac{\partial^{k-1} V}{\partial \bar{z}^{k-1}}\right), \quad (k = 2,3,\dots,n).$$

Equation (1) is investigated for $\alpha = 0$, $f_j(\varphi) \equiv 0$, $(j = 2,3,\dots,n)$, $f_1(\varphi) \equiv \text{const} \neq 0$ in [1]. For $\alpha = 0$, there is no singular line. This case is investigated for $f_j(\varphi) \equiv 0$, $(j = 3,4,\dots,n)$, $f_2(\varphi) \equiv \text{const} \neq 0$ in [2], and for $f_j(\varphi) \equiv 0$, $(j = 4,5,\dots,n)$, $f_3(\varphi) \equiv \text{const} \neq 0$ in [3]. In [4] the Cauchy problem and in [5] the Robin problem for (1) with $\alpha = 0$, $f_j(\varphi) \equiv 0$, $(j = 5,6,\dots,n)$, $f_4(\varphi) \equiv \text{const} \neq 0$ are solved.