Received: 14 October 2017 | Revised: 8 November 2017 | Accepted: 10 November 2017

DOI: 10.1002/ctpp.201700144

REVIEW ARTICLE

Contributions to Plasma Physics

Classical ion–grain scattering in plasmas: Image force correction

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Funding Information
This research was supported by the Ministry of Education and Science of the Republic of Kazakhstan, 2018–2020.

Corrections to the classical ion–grain scattering and capture cross-sections due to polarization charges on the dust particle, which acts as an image charge, have been obtained for a low-charged grain (with the charge number $Z = 10$). The trajectory of the ion around the dust particle is used for visual illustration of the influence of the grain polarization on scattering. The correction to the scattering cross-section due to the image force can be as large as 25% in the strong ion–grain coupling regime and up to 10% in the moderately coupled case. The correction to the capture cross-section turns out to be nearly constant (∼14%) for moderate as well as strong ion–grain coupling.

KEYWORDS

collection cross-section, complex plasma, polarization of the dust particle, scattering cross-section

1 | INTRODUCTION

A complex (dusty) plasma consists of electrons, ions, atoms, and charged dust particles (grains). The dust particle can have a size varying from several tens of nanometres to several micrometres. This allows investigation of the dynamics of an individual dust particle by visual methods (like video recording). The dust particle placed in a plasma becomes charged; therefore it is necessary to study the charging process and processes associated with the transfer of momentum due to elastic and inelastic collisions of plasma particles with the dust particle. There is a great variety of experimental data, theoretical models, and computer simulations of the charging processes of dust particles (see, e.g., Refs. 1–3) and the static and dynamic properties of dusty plasmas.[(4–12)] It have been found that charged dust particles can substantially alter the plasma properties. Using the Boltzmann equation,[(13–15)] it was shown that the presence of dust particles significantly changes the properties of the surrounding plasma, due to the dissipation of energy in the volume and absorption of electrons and ions on the surface of dust particles. The influence of dust particles on a plasma depends on the number density of dust grains and on the ion, electron–dust particle scattering (collection) cross-section. The momentum transfer between the various components plays an extremely important role in complex plasmas. The momentum transfer from ions to a charged dust particle plays an important role in such processes as the spatial arrangement of dust particles,[16] the rotation of dust structures in the presence of an external magnetic field,[17] the process of diffusion of non-interacting Brownian dust particles,[18] dust particle wave dispersion,[19] formation of a void,[20] etc. Usually, the impact of the image force is neglected. Therefore, the goal of this work is to study the correction to scattering and collection cross-sections due to the image force.[(21,22)]

The scattering process is characterized by the following dimensionless parameters: the coupling parameter $\beta = e^2 Z/mv^2\lambda$ (where m is the ion mass, v is the initial ion velocity (before collision)) and the radius of the dust particle normalized to the Debye length $\alpha = a/\lambda$. In the experiments involving a gas discharge with an ion temperature ∼300 K and electron temperature ∼1–3 eV, the ion–grain coupling parameter is in the range $\beta \sim 1$–30 and $Z \sim 10^3 - 10^4$. For a dust particle with such a large charge number, the effect of the image force on the scattering cross-section can be neglected.[23] However, in the experiments at cryogenic conditions,[(24–26)] Z is of the order of 10 and the image force becomes important. In cryogenic dusty plasmas, the neutral atom temperature is in the range of 5–100 K, and corresponding ion–grain coupling parameter can be estimated as $\beta \sim 1$–100. Moreover, the dynamics of the dust particle with a low charge can be of interest in the study of gases with a low

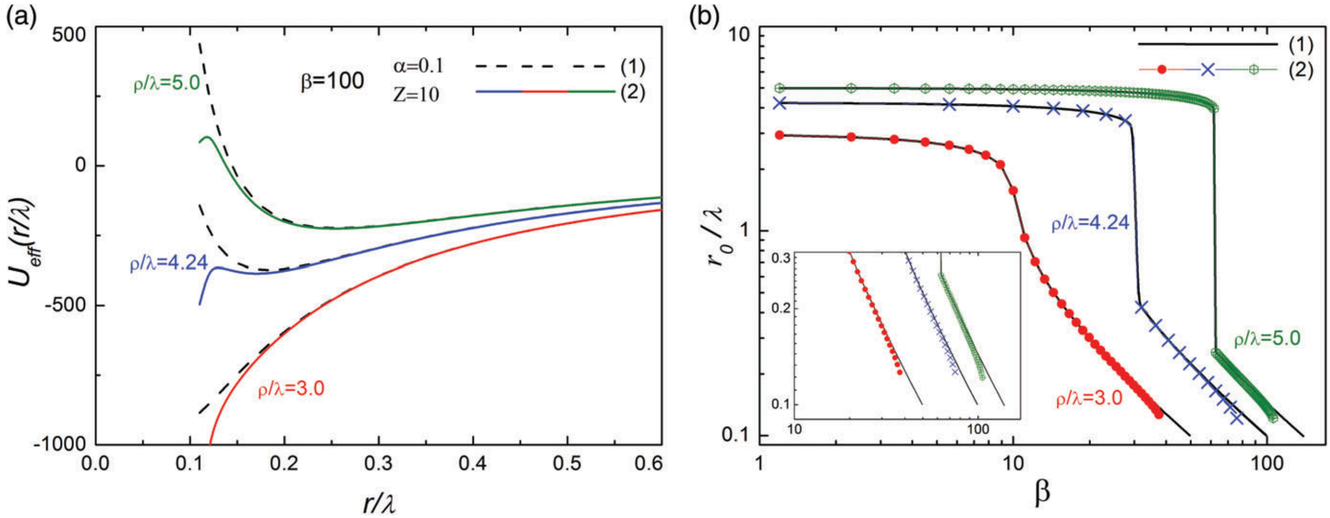

FIGURE 1 (a) Effective potential for the radial motion of ions in the field of the dust particle. (b) Distance of minimum approach, r_0/λ, as a function of the coupling parameter β. The results were obtained using the Yukawa potential (line 1) and the interaction potential (1) (line 2)

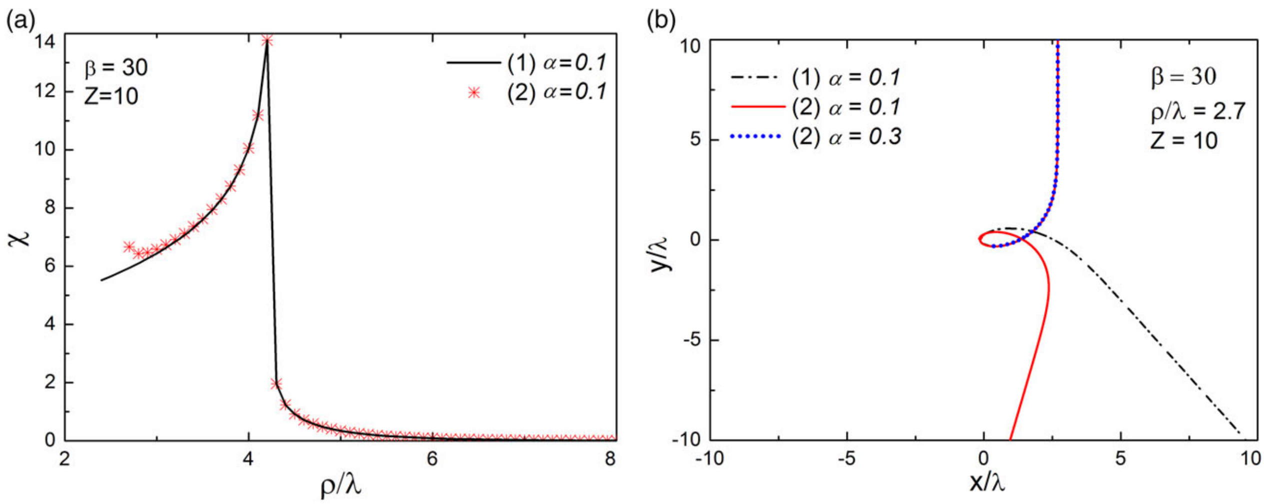

FIGURE 2 (a) Scattering angle obtained using the Yukawa potential (line 1) and the interaction potential (1) (line 2). (b) Ion trajectories during collision with a grain at $\alpha = 0.1$, 0.3; $Z = 10$; $\beta = 30$. The scattering centre (dust particle) is located at $x = 0$, $y = 0$

density and a low degree of ionization, or in the presence of a relatively weak radiation in the interstellar space. Therefore, in this paper, mainly motivated by the experiments on cryogenic complex plasmas,[(24,27)] we consider $Z = 10$ and $1 \leq \beta \leq 100$.

2 | SCATTERING CROSS-SECTION

In this work, the effect of the dust particle polarization on the scattering of a positively charged ion on the negatively charged grain is considered on the basis of the following ion–grain interaction potential:[23]

$$U(r) = -\left[\frac{e^2 Z}{r} + \frac{\xi e^2 a^3}{2r^2(r^2 - a^2)}\right] \exp(-r k_D), \tag{1}$$

where k_D is the inverse screening length (equal to the inverse value of the Debye length λ), a is the dust particle radius, and $\xi = 1$ for a metal grain and $\xi = (\varepsilon_d - 1)/(\varepsilon_d + 1)$ for a dielectric grain (further we take $\xi = 1$).

The first term on the rhs of Equation (1) is the familiar Yukawa (Debye) potential, and the second term is the correction for the pair interaction caused by the image force. In the limit $k_D \to 0$, the second term in the brackets on the rhs of Equation (1) is just the image force describing the interaction of the isolated spherical dust particle with the scattering ion.[21]

The classical scattering angle for two particles with masses m_1, m_2 and with the interaction potential $U(r)$ for a given impact parameter ρ is equal to

$$\chi(\rho) = |\pi - 2\varphi(\rho)|, \tag{2}$$

where

$$\varphi(\rho) = \rho \int_{r_0}^{\infty} \frac{dr}{r^2\sqrt{1 - U_{\text{eff}}(r,\rho)}}, \tag{3}$$

and the effective interaction energy U_{eff} in units of kinetic energy of a projectile $E = mv^2/2$ taking into account the centrifugal force (appearing as the result of the angular momentum conservation law) reads

$$U_{\text{eff}}(r,\rho) = \frac{\rho^2}{r^2} + \frac{2U(r)}{mv^2}. \tag{4}$$

In Equation (3), r_0 corresponds to the distance of minimum approach at the given ρ and is obtained from the equation $U_{\text{eff}}(r_0, \rho) = 1$.

The elastic scattering cross-section can be obtained using the scattering angle $\chi(\rho)$ in the standard way[28]:

$$\sigma_s = 2\pi \int_0^{\infty} (1 - \cos\chi(\rho))\rho d\rho, \tag{5}$$

where calculations must be performed taking into account the physical condition $r_0 > a$.

The collection cross-section is calculated according to the following formula[29]:

$$\sigma_c = \pi\rho_c^2. \tag{6}$$

The collection of ions is formed when the impact parameter of an ion is smaller than the capture impact parameter ρ_c, which is equal to $\rho_c = a\sqrt{1 + 2\beta(\lambda/a)}$ for the Yukawa interaction potential. In the case of the screened potential (1), the capture impact parameter is obtained from the condition that the distance of the closest approach r_0 is smaller than the radius of the dust particle a.

3 | RESULTS AND DISCUSSION

In Figure 1a, we show the effective potentials (4) for the radial motion of ions in the field of the dust particle obtained using the Yukawa potential and the interaction potential (1) at $\alpha = 0.1$, $Z = 10$, $\beta = 100$ and different impact parameters ρ/λ. As can be seen, the polarization effect is important at short distances near the dust particle surface. For the considered values of the impact parameter, the deviation from the Yukawa result appears at $a < r < 0.2\lambda$. The distance of minimum (closest) approach obtained using the interaction potential (1) is shown in Figure 1b in comparison with that calculated using the Yukawa potential. In Figure 1b, the values of the distance of minimum (closest) approach are shown for the case of scattered ions, i.e., escaped ions. Ions with r_0 smaller than that shown in Figure 1b are collected (captured) by the charged dust particle. The values of r_0 corresponding to the captured ions are larger for the case when the image force is taken into account in comparison with the

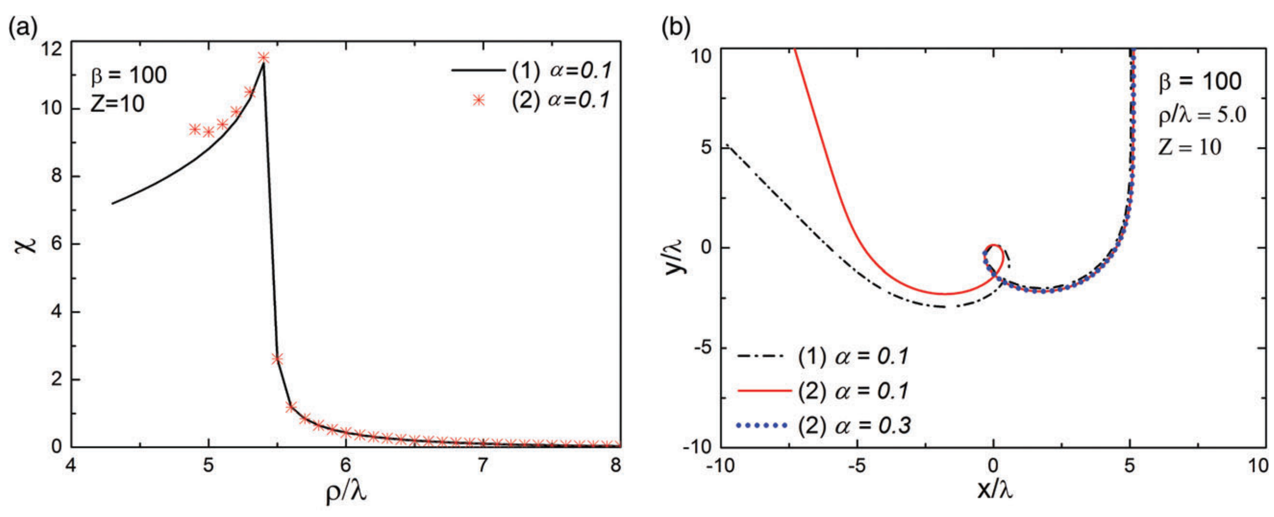

FIGURE 3 (a) Scattering angle obtained using the Yukawa potential (line 1) and the interaction potential (1) (line 2). (b) Ion trajectories at $\alpha = 0.1, 0.3$; $Z = 10$; $\beta = 100$. The scattering centre (charged grain) is located at $x = 0$, $y = 0$

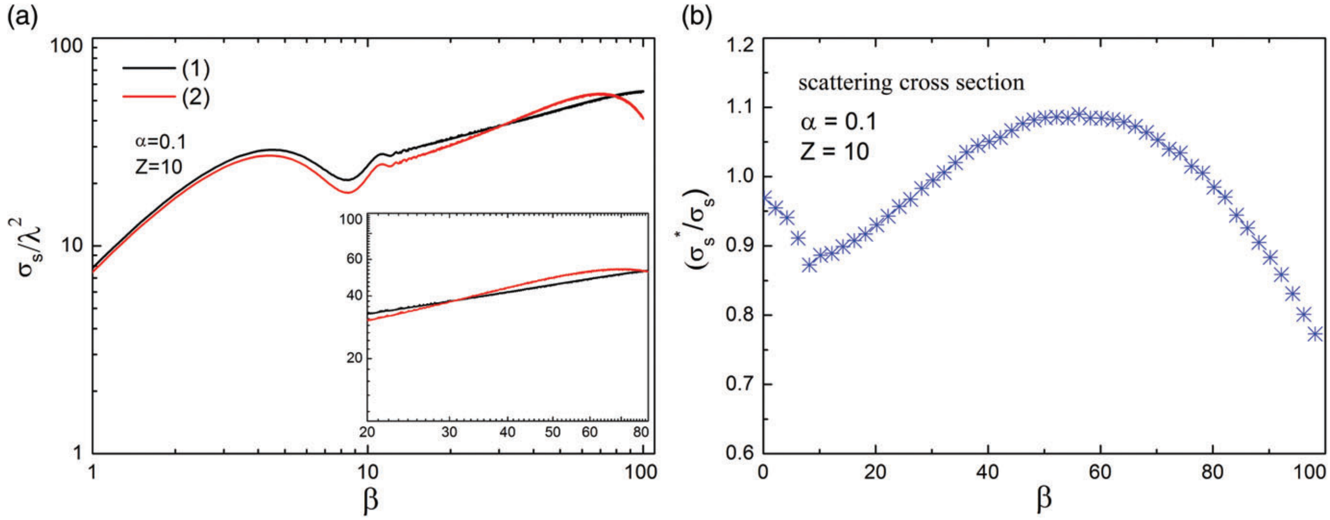

FIGURE 4 (a) Scattering cross-section obtained using the Yukawa potential (line 1) and the interaction potential (1) (line 2). (b) Ratio of scattering cross-sections based on the interaction potential (1) to the Yukawa potential

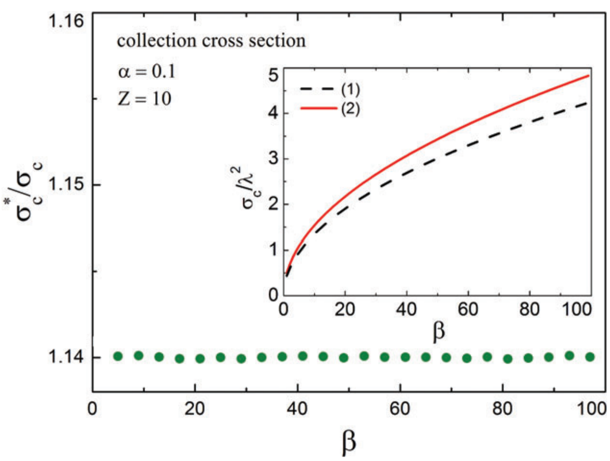

FIGURE 5 Ratio of the collection cross-sections obtained taking into account the image force correction, using the interaction potential (1), to that of the Yukawa potential

case when it is neglected. Therefore, the action of the image force results in a larger effective capture (absorption) radius of ions by the dust particle, as shown below.

The values of the scattering angle for $\beta = 30$ and $\beta = 100$ are shown in Figures 2a and 3a, respectively. The solid line shows the scattering angle obtained using the Yukawa potential, for the discussions of the non-monotonic behaviour around $\rho = 4.24$ we refer the reader to Ref. 30. At small impact parameters, polarization of the dust (the image force) leads to the deviation of the scattering angle from the Yukawa result. The corresponding ion trajectories around the dust particle are presented in Figures 2b and 3b. As one can see, when polarization is taken into account, the scattering angle is larger in the case of $\alpha = 0.1$, and when the dust particle size $\alpha = 0.3$ increases, the ion is captured (absorbed) by the dust particle.

The scattering cross-sections obtained using the Yukawa potential and the interaction potential (1), as well as their ratio, are shown in Figure 4. From the figure we can see that the effect of polarization of the dust particle (the image force) can both increase (at $30 < \beta < 80$) and decrease (at $1 < \beta < 30$ and $80 < \beta < 100$) the scattering cross-section. The correction to the scattering cross-section is up to 25% in the strong ion–grain coupling regime ($\beta > 80$) and up to 10% in the case $1 < \beta < 80$.

The ratio of the collection cross-section, which was calculated using the interaction potential (1), to that of the Yukawa potential[29] is shown in Figure 5. Interestingly, it is seen that the image force correction is ∼ 14% for all considered values of the coupling parameter, $1 < \beta < 100$.

4 | CONCLUSIONS

In this work, corrections to the ion–grain scattering and absorption (collection) cross-sections due to the image force (dust polarization) were presented for the case $Z = 10$ and $a/\lambda = 0.1$. The correction to the collection cross section is relatively weak, but surprisingly nearly constant (∼14%) for all considered values of the ion–grain coupling parameter, $1 < \beta < 100$. In contrast, the correction to the scattering cross-section due to the image force is a non-monotonic function of the coupling parameter β and can be significant for certain values of β (up to 25% at $\beta \simeq 100$).

The parameters of the calculations refer to cryogenic dusty plasmas. Cryogenic dusty plasmas are the emerging sub-field of complex plasmas and, despite their importance from the prospect of their application, are purely understood (e.g., see discussions in Ref. 31). Obtained results clearly show that the effect of the image force (polarization of a finite sized dust particle) is important for understanding the dynamics of grains in cryogenic dusty plasmas.

ACKNOWLEDGMENT

This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan under Grant (2018–2020).

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How to cite this article: Bastykova NK, Kodanova SK, Ramazanov TS and Moldabekov ZA. Classical ion–grain scattering in plasmas: Image force correction, *Contributions to Plasma Physics* 2017. https://doi.org/10.1002/ctpp.201700144.