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PROBLEM REALIZATION FOR CLASS NONLINEAR DETERMINISTIC SYSTEM OF POLYNOMIAL TYPE

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Осы жұмыста сызықты емес сингулярлық детерминандық жүйе сыныбы қарастырылады. Р.С.Судаковтың жалған–жартылай матрица негізінде мәселені шешудің жаңа жолы қарастырылған.

В данной работе рассматривается класс нелинейных сингулярных детерминированных систем. Предложен новый подход для решения проблемы реализации на основе псевдо –полуобратных матриц Р.С.Судакова.

In the device pseudo invers Moore– Penrose's matrixes is presented. In definition is given to a floor of the return matrixes and S.L.Sobolev's theorem for the decision of systems of the algebraic equations which was also used in. In the theory pseudo for the first time was submitted to a floor of the return matrixes of R. S. Sudakov and her application to problems of an assessment of reliability of systems. In the remark is made that pseudo the return matrixes it is difficult to use the theory in a number of tasks from linear algebra. In [1-4] definition of matrixes of Drazin, a nilpotent class of matrixes containing in the structure is given and their annex to the decision of systems of the nonlinear singular differential equations with use of the theorem of S.L.Sobolev is given. It is necessary to notice that the class of matrixes of Drazin is very narrow class of matrixes that is their properties of a commutatively, but some remarks from for pseudo invers matrixes here nevertheless were allowed. From [5] came a great idea "to formulate the realization theory of stochastic processes in the same natural way as it is done in the case of deterministic systems." In this direction, one of the most profound studies of linear discrete stochastic implementation problem was done in. The first stage of the algorithm for solving this problem is to solve the well-known problem of linear deterministic implementation for the case when parameters are set in 'weighting function." In the second step we solve the problem of spectral factorization, which completes by solution of algebraic Riccati's; equation. You can specify a number of papers have used this idea. However, in the above-mentioned studies, for example, was not considered a concept of stochastic systems state, i.e. no rigorous mathematical application of stochastic analysis has been done.

State Problem/Consider.

Class of Nonlinear Deterministic System

$$\dot{x}(t) = f(x(t)) + g(x(t))U(t), \qquad x(0) = x_0$$
$$\dot{x}(t) = f(x(t)) + g(x(t))U(t), \qquad x(0) = x_0 \qquad (1)$$

$y(t) = h[x(t)];$ $y(t) = h[x(t)];$
(2)

where $x(t) \in R^n$ $x(t) \in R^n$

$$f(x)=\sum_{i=1}^{n} A_i x^{[i]}, \qquad g(t)=\sum_{j=1}^{n} B_j x^{[j]}. \tag{3}$$

$$h(x)=\sum_{k=1}^{p} C_k x^{[k]},$$

$x^{[i]}=x\otimes x^{i-1}$ $x^{[i]}=x\otimes x^{i-1}$ - Kronecer Product.

Problem 1. We must construct of Volterra's series for system (1) – (3).

Solution of Problem 1. We can give following theorem.

Theorem 1. For Nonlinear Deterministic System (1) – (2) there exists equivalent singular bilinear systems.

$$E\cdot x(x)=Ax(t)+BxU(t), \qquad x(0)=x_0, \tag{4}$$

$$y(t)=Cx(t), \tag{5}$$

Where det (E)=0, $\tilde{x}=[x,x^{[2]},\cdots,x^{[n]}]^T$ $\tilde{x}=[x,x^{[2]},\cdots,x^{[n]}]^T$

$$E=\begin{pmatrix}1&0&0&\cdots&0\\0&1&0&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&1&\cdots&0\\0&0&0&\cdots&0\end{pmatrix}, \qquad \tilde{A}=\begin{pmatrix}A_1&A_2&\cdots&A_{n-1}&A_n\\0&A_{22}^{[2]}&\cdots&A_{2n-1}^{[n-1]}&A_{2n}^{[n]}\\\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&\cdots&A_{n-1,n-1}^{[n-1]}&A_{n-1,n}^{[n]}\\0&0&0&\cdots&0\end{pmatrix},$$

$$\tilde{B}=\begin{pmatrix}B_1&B_2&\cdots&B_{n-1}&B_n\\0&B_{22}^{[2]}&\cdots&B_{2n-1}^{[n-1]}&B_{2n}^{[n]}\\\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&\cdots&B_{n-1,n-1}^{[n-1]}&B_{n-1,n}^{[n]}\\0&0&0&\cdots&0\end{pmatrix}, \tag{6}$$

Proof: Applying formulas

$$\frac{d}{dt}\tilde{x}\otimes\tilde{x}=\frac{\widetilde{dx}}{dt}\,\tilde{x}\,\frac{\otimes\tilde{x}\otimes\widetilde{dx}}{dt},$$
$$\frac{d}{dt}\tilde{x}^{[h]}=\frac{\tilde{x}\otimes\widetilde{dx}^{[h-1]}}{dt}+\frac{\widetilde{dx}}{dt}\otimes\tilde{x}^{[h-1]}, \tag{7}$$

and

$$\bar{b}\otimes\bar{a}=c_{r,n}^{T}\left(\bar{a}\otimes\bar{b}\right). \tag{8}$$

$$B\otimes A=c_{r,n}^{T}(A\otimes B)C_{s,m},$$

where vectors $\bar{a}\in R^{r}$, $\bar{b}\in R^{n}$, matrixes $A\in R^{rxs}$, $B\in R^{nxm}$,

We obtain (4), (5) and (6).

Theorem2. For Singular Bilinear Systems (4) – (6) there exists equivalent Bilinear Systems.

$$\dot{x}(t) = A_1\tilde{x}(t) + B_1\tilde{x}U(t) + B_2V(\xi), \qquad (9)$$

$$y(t) = Cx(t), \qquad (10)$$

where

$$A_1 = E^c \cdot A, \qquad B_1 = E^c \cdot B, \qquad B_2 = I - E^c \cdot E, \qquad (11)$$

$E^c\, E^c$ - pseudo-semi- inverse Sudakov's; matrix,

$V(\xi)$ – constant vector,

If

$$E \cdot E^c(Ax(t) + B_1xU(t)) = Ax(t) + BxU(t) \qquad (12)$$

Proff: Suppose that condition (12) is true. Multiplying of both parts equation (9) in left on matrix E, we have

$$Ex(t) = E(A_1x(t) + B_1x(t)U(t)) + E(I - E^cE) \qquad (13)$$

or

$$Ex(t) = E \cdot E^c(Ax(t) + Bx(t)U(t)) + E - EE^cE$$

since

$$E = EE^cE,$$

then

$$Ex(t) = Ax(t) + Bx(t)U(t)$$

Consequently, we obtain equation (4).

Theorem3. For Singular Bilinear Systems (9) – (11) there exists Volterra series.

$$y(t) = \sum_{k=0}^{t} \int_0^t \int_0^{\theta_1} \cdots \int_0^{\theta_{k-1}} W_k(t,\theta_1,\cdots,\theta_k)U(t)dt + \int_0^t W_1^*(t-\theta_1)V(\xi)d\theta_1, \qquad (14)$$

where

$$W_k(t_1\theta_1,\cdots,\theta_k) = Ce^{A(t-\theta_1)} \cdot B_1e^{A(\theta_1-\theta_2)} \cdot B \dots e^{A(\theta_{k-1}-\theta_k)}\overline{x_0},$$

$$W_1^*(t-\theta_1) = Ce^{A(t-\theta_1)} \cdot B_2$$

Theorem4. Volterra series (14)-(15) converging uniformly on to solution of (4)-(5) if pair (A, B) is nilpotent pair from Lie algebra.
Proff: See Brokett R.W. "Volterra series and Geometric Control Theory" 1971 year, Preprints IFAC, Philadelphia,USA, using the Baker – Cambell – Hausdorf formula and nilpotent property of Lie algebras.

PART TWO.

Problem 2. (Problem realization).

Let we know I/O Map (14)-(15). We must defind matrixes A_1, B_1, B_2 A_1, B_1, B_2 for systems (9)-(10).

Solution of Problem Realization.

Realization algorithms. Let

$$n^i = rank S'_M M \quad (16)$$

where $S'_M M$ $S'_M M$ - Isidori's matrix

and find matrix P and Q such that

$$Q \cdot P = S'_M M \quad (17)$$

Write the bilinear realization

$$A = (Q^T \cdot Q)^{-1} Q^T S^1_{MM} P^T (P \cdot P^T)^{-1}$$

$$B_1 = \left(Q^T \cdot Q\right)^{-1} Q^T S^2_{MM} P^T \left(P \cdot P^T\right)^{-1}$$

(18)

C – First row of Q.

Applying B. Ho algorithm, we obtain matrix B_2 B_2. If i=1 formula (16), we obtain algorithm by names Alberto Isidory. Consequently, we obtain block algorithm A. Isidory. Using formulas (11) we have A, B and E^c. E^c.

Example 1.

$$\dot{x} = a_1 x + a_2 x^2 + (b_1 x + b_2 x^2) U(t), \dot{x} = a_1 x + a_2 x^2 + (b_1 x + b_2 x^2) U(t),$$
(19)

$$\dot{x} = 2x\dot{x} = 2x(a_1 x + a_2 x^2) + 2x(b_1 x + b_2 x^2) U(t) = \quad (20)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{x}^2 \\ \dot{x}^3 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & 0 \\ 0 & 2a_1 & 2a_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ x^2 \\ x^3 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 & 0 \\ 0 & 2b_1 & 2b_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ x^2 \\ x^3 \end{pmatrix} U(t),$$

(21)

where

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \det(E) = 0. \; E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \det(E) = 0.$$
(22)

Applying theorem 2, we have

$$\dot{\tilde{x}} = A_1\tilde{x} + B_1\tilde{x}U(t) + B_2V(\xi), \dot{\tilde{x}} = A_1\tilde{x} + B_1\tilde{x}U(t) + B_2V(\xi),$$
(23)

$$\tilde{x} = \begin{pmatrix} \dot{x} \\ \dot{x}^2 \\ \dot{x}^3 \end{pmatrix}, \qquad A_1 = \begin{pmatrix} a_1 & a_2 & 0 \\ 0 & 2a_1 & 2a_2 \\ 0 & 0 & 0 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} b_1 & b_2 & 0 \\ 0 & 2b_1 & 2b_2 \\ 0 & 0 & 0 \end{pmatrix} \tag{24}$$

$$E^C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ [6-14].

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