Mathematical and Computer Modeling of the Stability of Complex Electric Power Systems

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Abstract

This article discusses the development and study of mathematical model of complex power systems for the global asymptotic stability problems. The conditions have been obtained for the global asymptotic stability of nonlinear control systems. Control actions that ensure stabilization of complex electric power systems have been found. The software package of dynamic study of complex electric power systems has been developed in Visual Studio using the programming language C-Sharp.

1 Introduction

The industrial development of modern society leads to a constant increase in electricity consumption. Complex electric power systems are created to meet these growing needs.

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Under current conditions of Kazakhstan's power generating industry development is at a high level of wearing out and insufficiently high rates of equipment modernization, this question is even more acute [Jacobson & Ycesan, 2008]. Besides Kazakhstan's power industry modernization necessity, it is required to estimate effectiveness of modern equipment use and conditions of its normal operation.

By their normal functioning electric power systems (EPS) provide operation of industry, transport, population household - the entire life of the country. Development of power systems is moving along the path of creating large energy associations, in some cases covering entire continents, such systems include large number of generators, the emerging tendency of transferring generators to the reactive power consumption mode in order to normalize voltage levels in the network has complicated the problem of ensuring the EPS stability [Gelig et al., 1978]-[Kalman, 1963]. All these negative factors caused frequent violations of the vibrational stability of EPS, emphasizing the relevance and practical importance of solving this problem. These circumstances required the improvement of research methods and in-depth study of EPS's dynamic properties. At this stage, the development of highly efficient methods for numerical solution of global asymptotic stability problems, T-controllability, optimality, and carrying out wide-ranging computational studies were of great importance.

The importance of the problem ensuring stable operation of electric power systems is confirmed by a large number of developments, both domestic and foreign scientists. We can note the works of such scientists as P. Kalman, P. Anderson, A. Fouad [Anderson & Fouad, 2002], L.E. Jones, J. Klir, S. Arimota , etc., and also scientists from near abroad L.Y. Anapolsky, A.A. Yanko-Tryniski, I.I. Blekhman, A.H. Gelig [Gelig et al., 1978] etc. and from Kazakhstan S.A. Aisagaliyev [Aisagaliev & Kalimoldayev, 2013], S.A. Aipanov, M.N. Kalimoldayev [Kalimoldayev et al., 2014], M.T. Jenaliev, G.A. Leonov, V.M. Matrosov, V.A. Yakubovich, K. Toktybakiev, V.A. Korotkov, L. Kopbosyn, etc.

2 Formulation of the Problem

The system of nonlinear differential equations:

$$\frac{dx}{dt} = A(t)\,x + D(t)\,u + f(t,u,x)\,,\ t \in [t_0,t_1]\,, x(t_0) = x_0$$

$$A = \begin{bmatrix} 0 & S_i & 0^*_{n_i} \\ 0 & -K_i & c^*_i \\ 0_{n_i} & b_i & \overline{A}_i \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ q_i \end{bmatrix}$$

$$x = \begin{bmatrix} \delta_i \\ S_i \\ \overline{x_i} \end{bmatrix},\ \overline{f}(x) = \begin{bmatrix} 0 \\ -f_i(\delta_i) - \psi_i(\delta) \\ 0_{n_i} \end{bmatrix}$$

Consider the general mathematical model for electric power systems:

$$\frac{d\delta_i}{dt} = S_i,$$

$$\frac{dS_i}{dt} = (W_i - K_i S_i - f_i(\delta_i)) - \psi_i(\delta_{i*})\,, \qquad w_i = C^*_i x_i, \tag{1}$$

$$\frac{dx_i}{dt} = A_i x_i + q_i S_i + b_i u_i + R_i(S_i, x_i)\,, i = \overline{1,l}, \tag{2}$$

where function

$$\psi_i(\delta_{i*}) = \sum_{\substack{k=1 \\ k \neq i}}^{l} P_{ik}(\delta_{ik})\,, \qquad \delta_{ik} = \delta_i - \delta_k \tag{3}$$

where δ_i – the angular coordinate; S_i – angular velocity; x_i, n_i – the state vector of the regulator; w_i - the control action of the regulator; $K_i > 0$ - damping factor; c_i, q_i, b_i – constant n_i – dimensional vectors; A_i – constant $(n_i \times n_i)$ matrix; u_i – control of the feedback type. The symbol (*) denotes transposition. Second order differential equations (1) describe the processes in the object of control, and the vector differential equation (2) determines the state of the regulator of the i-th isolated subsystem, in which the phase trajectories tend to the particular stable equilibrium position.

Consider the global asymptotic stability of the coupled system with many angular coordinates in the case when

$$R_i\left(S_i, x_i\right) = e_i \phi_i\left(\sigma_i\right), \ \sigma_i = g_i^* x_i + \ \gamma_i S_i, \quad i = \overline{1, l}, \tag{4}$$

where $g_i e_i$- constants, n_i- vectors, γ_i- scalar constant. (4) change shows the Euclidean space and consist of x and S. The system (1)-(3) takes the form:

$$\frac{d\delta_i}{dt} = S_i, \frac{dS_i}{dt} = w_i - K_i S_i - f_i\left(\delta_i\right) - \psi_i\left(\delta_{i^*}\right), \ w_i = C_i^* x_i, \tag{5}$$

$$\frac{dx_i}{dt} = A_i x_i + q_i S_i + b_i u_i + e_i \phi_i\left(\sigma_i\right), \ i = \overline{1, l} \tag{6}$$

or in vector-matrix form

$$\frac{d\delta}{dt} = S, \frac{dS}{dt} = w - KS - f\left(\delta\right) - \psi\left(\delta_*\right), \ w = C^* x, \tag{7}$$

$$\frac{dx}{dt} = Ax + qS + bu + e\phi\left(\sigma\right), \ \sigma = g^* x + \gamma S, \tag{8}$$

where $e = colon\left(e_1, \ldots, e_l\right), \gamma = colon\left(\gamma_1, \ldots, \gamma_l\right), \ g = diag\left\{g_1, \ldots, g_l\right\}$.

Characteristics of nonlinear elements $\phi_{i_i}\left(\sigma_i\right)$ are continuous functions satisfying the conditions

$$0 \le \phi_{i_i}\left(\sigma_i\right)\sigma_i \le \rho_i \sigma_i^2, \quad \phi_i\left(0\right) = 0, \quad i = \overline{1, l}$$

$$\left(\forall \sigma_i \in \left(0, +\infty\right), \quad \forall \sigma_i \in R_i^1\right) \tag{9}$$

If there are inequalities for the function $\phi_{i_i}\left(\sigma_i\right)$:

$$\rho_{1i}\sigma_i^2 \le \phi_{i_i}\left(\sigma_i\right)\sigma_i \le \rho_{2i}\sigma_i^2,$$

$$\left(\rho_{1i}\sigma_i^2, \ \rho_{2i}\sigma_i^2 \in \left(-\infty, +\infty\right), \forall \sigma_i \in R_i^1\right),$$

after substitution $\overline{\phi}_i = \phi_i - \rho_{1i}\sigma_i$ it leads to considered case; where $\rho_i = \rho_{2i} - \rho_{1i}$. The differential equation (6) will be rewritten in the form

$$\frac{dx_i}{dt} = \overline{A_i} x_i + \overline{q_i} S_i + b_i u_i + e_i \overline{\phi_i}\left(\sigma_i\right), \quad i = \overline{1, l},$$

where $\overline{A_i} = A_i + \ \rho_{i_i} e_i g_i^*, \ \overline{q_i} = q_i + \rho_{1i}\gamma_i$, and nonlinearity $\overline{\phi_i}\left(\sigma_i\right)$ satisfies the condition

$$0 \le \phi_i\left(\sigma_i\right)\sigma_i \le \left(\rho_{2i} - \rho_{1i}\right)\sigma_i^2,$$

$$\left(\rho_{2i} - \rho_{1i} = \rho_i \in \left(0, +\infty\right), \ \forall \sigma_i \in R_i^1\right).$$

Constraint (9) is equivalent to the inequality

$$\phi_i\left(\sigma_i\right)\left(\sigma_i - \rho_i^{-1}\phi_i\left(\sigma_i\right)\right) \ge 0, \tag{10}$$

$$\left(\rho_i \in \left(0, +\infty\right), \ \forall \sigma_i \in R_i^1\right), i = \overline{1, l}.$$

The function

$$\psi_i\left(\sigma_i\right) = \int_0^{\sigma_i} \phi_i\left(\lambda\right) d\lambda, \quad i = \overline{1, l} \tag{11}$$

is positive semi-definite function.

Theorem 1. Let there exist scalars $D_i, \tau_i > 0$ such that is globally asymptotically stable

1. The phase system of the second order (1) is globally asymptotically stable (i.e. $D_i > (D_i)_{kp}$).
2. A should be Hurwitz matrix $\tilde{A}_i$
3. $(\widetilde{A}_i, G_i)$ – completely observable pair.
4. $(\widetilde{A}_i, Q_i)$ – completely controllable pair .

$$\overline{\overline{\Gamma}}_i > 0,\ det\left[2\overline{\overline{\Gamma}}_i - \chi_i^* h_i^* \widetilde{D}_i^{-1} h_i \chi_i\right] \neq 0\ \left(i = \overline{1,l}\right).$$

5. $\Gamma_i + Re\ W_i\,(j\omega) \geq 0\,(\forall\omega \in (-\infty, +\infty))\,,\ L_i \geq 0, \widetilde{D}_i > 0.$

Then control

$$\begin{array}{lcll} u_i & = & a_i^* x_i + \theta_i S_i + \overline{\varepsilon_i}?_i\,(\delta_i) + \frac{S_i \psi_i(\delta_{i^*})}{x_i^* H_i b_i} & \text{at } z_i \in \sum_i \\ u_i & \neq & -(b_i^* H_i b_i)^{-1}(b_i^* H_i A_i x_i + b_i^* H_i q_i S_i + b_i^* H_i e_i \phi_i(\sigma_i)) & \text{at } z_i \notin \sum_i, i = \overline{1,l} \end{array} \tag{12}$$

ensures global asymptotic stability of the system (8), (9).

3 Numerical Example

We will consider motion stabilization of electric power system when $x(t_1) = x_1$ (two synchronous generators). The equations (5) and (6) describe the operation of the system, and the equations (7) and (8) describe the state of the controller

$$\begin{array}{l} \frac{d\delta_1}{dt} = S_1, \\ \frac{dS_1}{dt} = c_1 x_1 - K_1 S_1 - f_1\,(\delta_1) - P_{12}, \end{array} \tag{13}$$

$$\begin{array}{l} \frac{d\delta_2}{dt} = S_2, \\ \frac{dS_2}{dt} = c_2 x_2 - K_2 S_2 - f_2\,(\delta_2) - P_{12}, \end{array} \tag{14}$$

$$\frac{dx_1}{dt} = A_1 x_1 + u_1, \tag{15}$$

$$\frac{dx_2}{dt} = A_2 x_2 + u_2, \tag{16}$$

where x_1, x_2 – phase variables; c_1, c_2 – scalars; u_1, u_2 – controls initial conditions:

$$\begin{array}{rr} \delta_{10} = 1.34; & \delta_{20} = 0.84 \\ S_{10} = 0.0; & S_{20} = 0.0; \\ x_{10} = 0.001; & x_{20} = 0.001; \end{array}$$

All conditions of the theorem 1 have been checked.

$$\frac{d\delta}{dt} = S, \quad \frac{dS}{dt} = w - KS - f\,(\delta) - \psi\,(\delta_*)\,,\ w = C^* x, \tag{17}$$

$$\frac{dx}{dt} = Ax + qS + bu + e\phi\,(\sigma)\,, \qquad \sigma = g^* x + \gamma S, \tag{18}$$

We use the 4-th order Euler and Runge-Kutta methods to obtain the numerical solutions of equation systems (13)-(16).

The results of the numerical solution are shown in figures (1)-(4). We use the 4-th order Adams-Bashford, Adams-Moulton and Runge-Kutta methods for more accurate results.

Adams-Bashford method:

$$\begin{array}{rcl} y_{n+4} & = & y_{n+3} + \frac{h}{24}\,(55 f(t_{n+3}, y_{n+3}) - 59 f(t_{n+2}, y_{n+2}) + \\ & & +37 f(t_{n+1}, y_{n+1}) - 9 f(t_n, y_n))\,, \quad \frac{251}{720} h^5(\eta). \end{array}$$

Adams–Moulton method:

$$\begin{aligned} y_{n+4} &= y_{n+3} + \frac{h}{24}\left(9f(t_{n+4}, y_{n+4}) + 19f(t_{n+3}, y_{n+3}) - \right. \\ &\left. -5f(t_{n+2}, y_{n+2}) + f(t_{n+1}, y_{n+1})\right), -\frac{19}{720}h^5(\eta). \end{aligned}$$

Runge-Kutta method:

$$y_{n+1} = y_n + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right),$$

Comparison of the used methods is shown below:

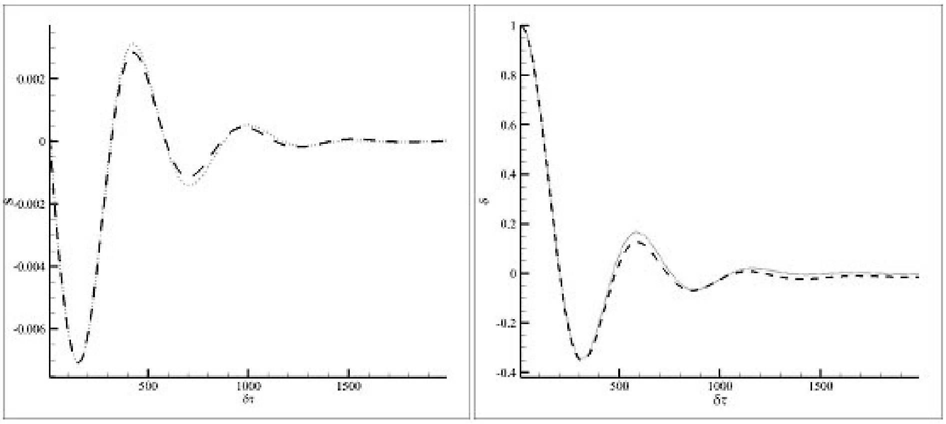

Figure 1: methods of Adams-Bashford (line - - -), Adams–Moulton (line ...) of the 4-th order a) time change of S ; b) time change of δ.

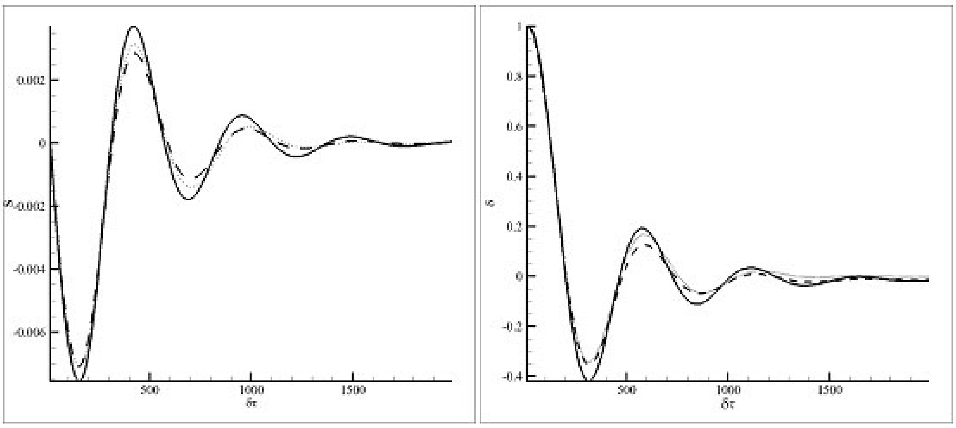

Figure 2: methods of Adams-Bashford (line- - -), Adams–Moulton (line ...) and Euler (line -) of the 4-th order a) time change of S ; b) time change of δ

The main purpose of using different methods for solving this problem is to decrease the amplitude over the same period of time. Since large values can affect the operation of the system and the life of the equipment. The fourth-order Euler method was used for comparison with other methods. As can be seen in Figure 2, the methods Adams-Bashford and Adams-Moulton show a smaller amplitude than the Euler method. Figure 1 shows the methods Adams-Bashford and Adams-Moulton, it follows that the method Adams-Bashford gives a good result. If compare the time for compilation, then the method Adams-Bashford also shows a better result than the rest. This is due to the fact that method Adams-Moulton is implicit. Figures 3 shows a comparison of Euler and Runge-Kutta. Figure 4 shows a comparison of the methods Adams-Bashford and Adams-Moulton with the Rung-Kutta

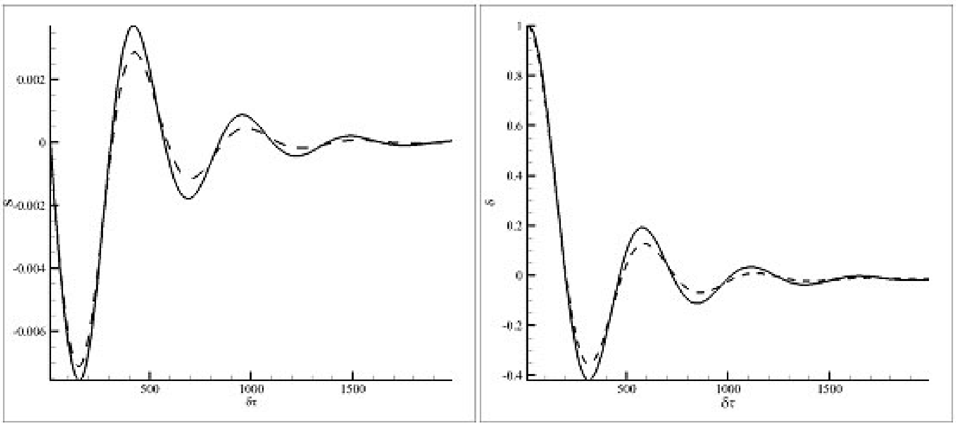

Figure 3: methods of Runge-Kutta (line - - -) and Euler (line -) of the 4-th order a) time change of S ; b) time change of δ

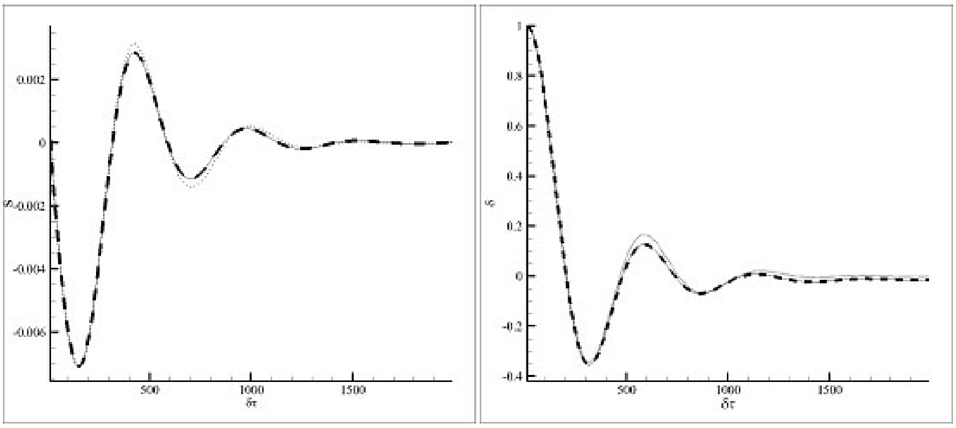

Figure 4: methods of Adams-Bashford (line - - -), Adams–Moulton (line ...) and Runge-Kutta (line-) of the 4-th order a) time change of S ; b) time change of δ .

According to the obtained results it is clear that the increase of more than 4 is not necessary, as they equally converge to zero.

For this task the Adams-Bashford and Runge-Kutta methods converge to zero faster than when using the method of Adams-Moulton. It allows to reduce time and speed up the process of determining emergency situation. Since the Adams-Moulton method is implicit and requires the solution of the "historical" values, which takes computation time.

The software package of dynamic study of complex electric power systems has been developed in Visual Studio using the programming language C-Sharp. A detailed analysis of the problem with definition and the identification of input and output information has been conducted.

Two types of testing have been carried out: functional and structural.

In functional testing the programs are verified the compliance behavior of the program to its external specification. The logic of the program is verified when a structural test.

The software products were initially created for the individual tasks of complex electrical power systems. Next, they were merged into a single set of programs. The 4-th order Adams-Bashford, Runge-Kutta and Adams-Moulton methods have been used when creating a software product. The figures (5-8) have shown the interface of software package.

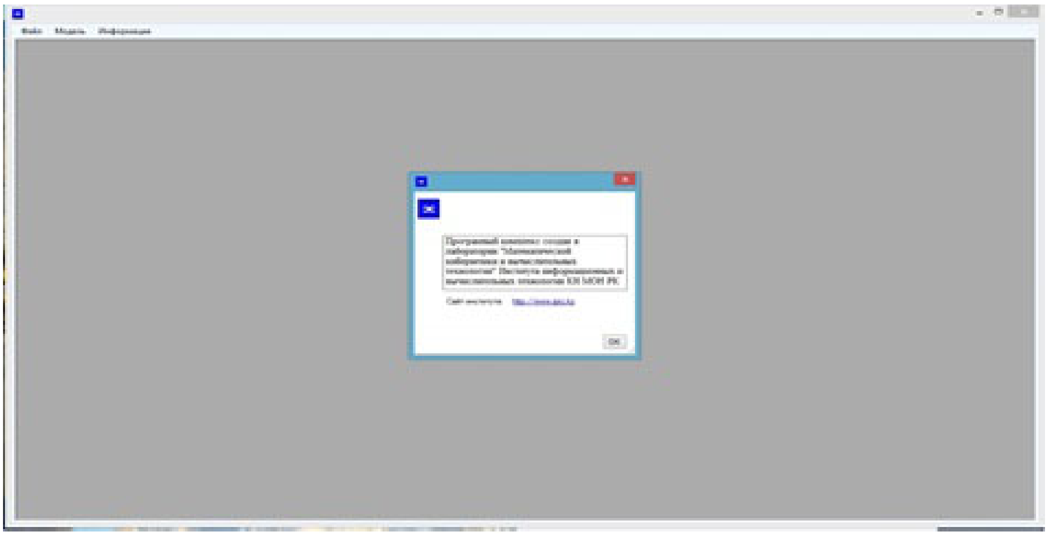

Figure 5:

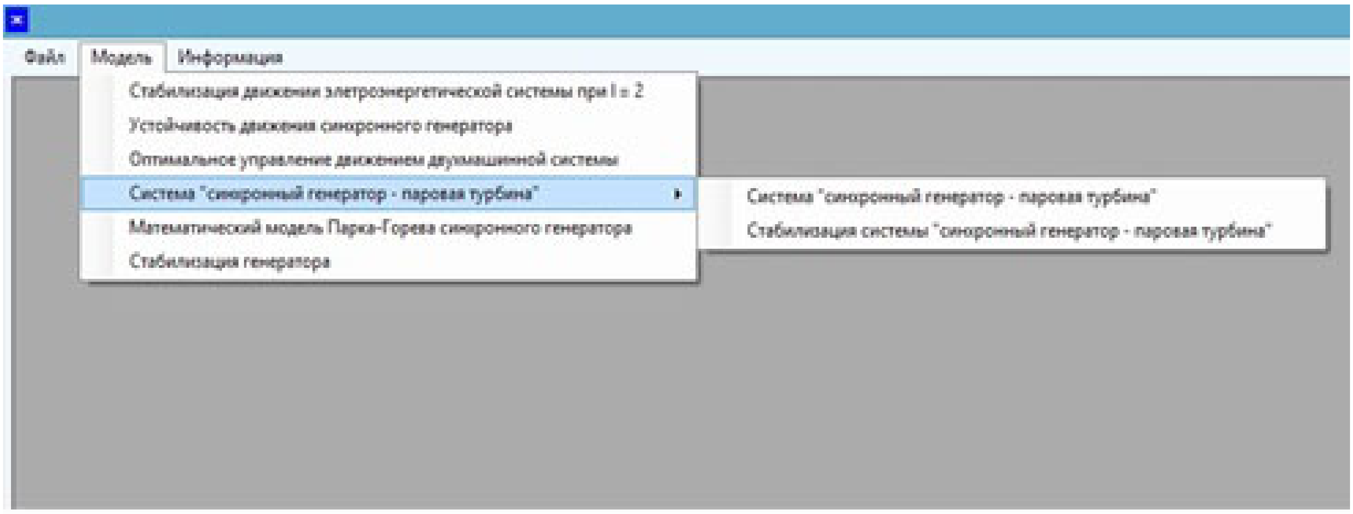

Figure 6: Main menu.

4 Conclusion

In this paper we solve the problem of global asymptotic stability of phase systems. According to the results obtained, it is clear that an increase in the order of more than four times is not necessary, since they converge to zero identically. For this problem, the Adams-Bashford and Runge-Kutte methods converge to zero faster than using the Adams-Moulton method. This allows you to reduce the time and speed up the process of determining the emergency situation. The software package of dynamic study of complex electric power systems has been developed in Visual Studio using the programming language C-Sharp.

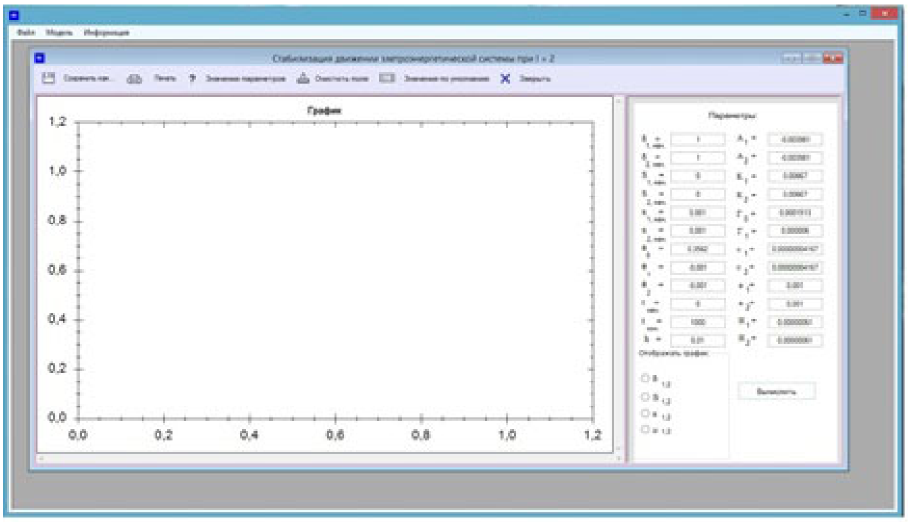

Figure 7: Numeric data entry on the selected method.

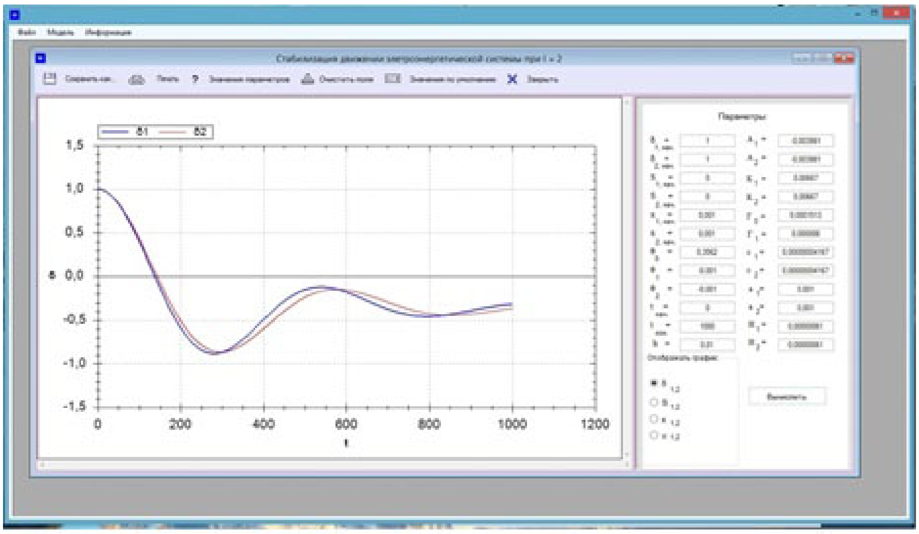

Figure 8: Illustration of graphics on the screen.

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