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Constructive method for solvability of Fredholm equation of the first kind

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Abstract. The solvability and construction of the general solution of the first kind Fredholm integral equation are among the insufficiently explored problems in mathematics. There are various approaches to solve this problem. We note the following methods for solving of ill-posed problem: regularization method, the method of successive approximations, the method of undetermined coefficients. The purpose of this work is to create a new method for solvability and construction of solution of the integral equation of the first kind. As it follows from the foregoing, the study of the solvability and construction of a solution of the Fredholm integral equation of the first kind is topical. A new method for studying of solvability and construction of a solution for Fredholm integral equation of the first kind is proposed. Solvability conditions and the construction method of an approximate solution of the integral Fredholm equation of the first kind are obtained.

Keywords: integral equation, solvability, construction of a solution, extreme problem, functional gradient, minimizing sequences.

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1 Introduction

Solving of the controllability problems of dynamical system [1,6,8], the mathematical theory of optimal processes [2,10,11], the boundary value problems of differential equations with phase and integral constraints [7,12,14] are reduced to solvability and construction of the general solution of the first kind integral equation

$$Ku = \int_{t_0}^{t_1} K(t,\tau)u(\tau)d\tau = f(t), \tag{1.1}$$

where $K(t,\tau)$ is a measurable function on the set $S_0 = \{(t,\tau) \in \mathbb{R}^2 \ / \ t_0 \le t \le t_1, \ \ t_0 \le \tau \le t_1\}$ and there exists an integral

$$P^2 = \int_{t_0}^{t_1}\int_{t_0}^{t_1} |K(t,\tau)|^2 dtd\tau < \infty,$$

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