Comparing Nontriviality for the Exponential Time Classes E and EXP

Timur Bakibayev

University of Heidelberg
Institut für Informatik
Im Neuenheimer Feld 294
D-69120 Heidelberg
Germany

Abstract. Lutz (1995) calls a set A *weakly complete* for a complexity class C if a *nonnegligible* part of C can be reduced to A (by a polynomial-time many-one reduction). For the exponential-time classes E = DTIME($2^{O(n)}$) and EXP = DTIME($2^{poly(n)}$), Lutz formalized this idea by introducing resource bounded measures on these classes and by saying that a subclass of E (EXP) is negligible if it has measure 0 in E (EXP).

We generalize Lutz's weak completeness notions for the exponential-time classes by calling a set A E-*nontrivial* if, for any $k \geq 1$, there is a set $B \in$ E $\setminus$ DTIME(2^{kn}) such that $B \leq_m^p A$, and by calling a set A EXP-*nontrivial* if, for any $k \geq 1$, there is a set $B \in$ EXP $\setminus$ DTIME(2^{n^k}) such that $B \leq_m^p A$.

As one can easily show, any E-complete set is weakly E-complete, any weakly E-complete set is E-nontrivial, and any E-nontrivial set is intractable but none of these implications can be reversed (and, similarly, for EXP in place of E).

While, for sets in E, E-completeness and EXP-completeness coincide, weak E-completeness is strictly stronger than weak EXP-completeness (Juedes and Lutz, Ambos-Spies, Terwijn and Zheng).

In case of the still weaker nontriviality notions we get the following independence result: For sets in E, neither E-nontriviality implies EXP-nontriviality nor EXP-nontriviality implies E-nontriviality. Moreover, there is a weakly EXP-complete set which is not E-nontrivial.

(This is joint work with Klaus Ambos-Spies.)