## CELMEC VIII

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## ABSTRACTS

OF THE POSTERS

## Canonical formalism for resonant returns

Close encounters between planets and asteroids can result in orbital resonance between the two bodies which will lead to a resonant return. Although approximate analytical models already exist, our aim is to introduce a Hamiltonian description of resonant returns. For a suitable subspace of the phase space, we build a chain of canonical transformations linking the encounter state before the first encounter to the state at the resonant return. The sequence of transformations is based on a two-body patched-conics approach. We check that our model gives good approximations by a comparison with a three-body evolution. A set of canonical coordinates that gives us the position of the small body on the target plane is also introduced. Finally, we describe the domain of our canonical transformation and its image at the second encounter.

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## Stability analysis in the Perturbed Robe's- Finite straight segment model under the effect of viscosity

In this Article, Robe-finite straight segment model is analysed under the effects of viscosity and perturbations in the Coriolis and centrifugal forces. We have taken the first primary as a rigid spherical shell filled with viscous, homogeneous incompressible fluid, and the second primary as a finite straight segment of length 2l. A third body, moving inside the bigger primary is a small solid sphere. We prove how the locations of equilibrium points are affected by the presence of perturbation in the centrifugal force. However, these remain unaffected by the viscosity and perturbation in the Coriolis force. The stability criteria for them are investigated and it has been observed that the stability of the collinear equilibrium points is affected by the viscosity and perturbation of the centrifugal force. It is prominently observed that the viscosity changes their nature of stability from being stable to asymptotically stable. The non collinear equilibrium points are unstable irrespective of the perturbation and viscosity.

Aiken Kosherbayeva<br>Al-Farabi Kazakh National University<br>Equations of secular perturbations of exoplanetary systems with variable masses

The study of dynamical evolution of exoplanetary systems is actual topic in astrodynamics and in celestial mechanics. For today, more than 5,000 confirmed exoplanets are known [1], and this list is growing rapidly. Researching of dynamics of exoplanets in the non-stationary stage of its formation gives us the opportunity to determine further evolutionary tracks. The influence of the variability of the masses of celestial bodies is explored on the dynamic evolution of planetary systems, considering that the masses of bodies change isotropically with different velocities. The problem of many bodies is considered in a relative coordinate system, with assuming that the most massive body - the parent star is located at the origin of this coordinate system. All $n$ bodies in the system will interact with each other according to Newton's law. Orbits of $n$ planets around the parent star are quasi-elliptical and we believe that they do not intersect. Bodies are considered spherically symmetrical with isotropically varying masses. We consider the laws of the masses to be known and arbitrary functions of time. Differential equations of motion of $n$ bodies in the relative coordinate system are given in the works [2-3]. The methods of canonical perturbation theory are used here, which developed on the basis of aperiodic motion over a quasi- canonical section [4] in analogues of the second Poincaré system of variables. The obtained canonical equations of perturbed motion [5] are most convenient for describing the dynamic evolution of planetary systems in the case when analogues of eccentricities and analogues of inclinations of the orbital plane are small enough. The non-resonant case is researched. The Wolfram Mathematica package is used in the expansion of perturbing functions into series. Since we are
interested in the evolution of orbital parameters over long periods of time, short-period perturbations associated with the orbital motion of bodies should be eliminated by averaging the perturbation functions by mean longitudes. As a result, we get the secular parts of perturbing functions. Secular perturbations of eccentric and oblique elements are defined as solutions of a system of 4 n linear differential equations. As an example, we consider the four-planet exosystem V1298 Tau (spectral type K0) [6] in the non-stationary stage of its evolution. To find secular perturbations, it will be necessary to solve a system of 16 linear nonautonomous differential equations. The obtained equations of secular perturbations are studied by the numerical method. Bibliography [1] https://exoplanets.nasa.gov/ [2] Minglibayev M.Zh., Kosherbayeva A.B. Differential equations of planetary systems // Reports of the National Academy of Sciences of the Republic of Kazakhstan, - 2020, -Vol.2(330). -P. 14-20, https://doi.org/10.32014/2020.2518-1483.26 [3] Minglibayev M.Zh., Kosherbayeva A.B. Equations of planetary systems motion // News of The National Academy of Sciences of the Republic of Kazakhstan. Physico-Mathematical Series, -2020, -Vol.6(334). -P. $53-60$, https://doi.org/10.32014/2020.2518-1726.97 . [4] Minglibayev M.Zh. Dynamics of gravitating bodies with variable masses and sizes [Dinamika gravitiruyushchikh tel s peremennymi massami i razmerami]. LAP LAMBERT Academic Publishing. -2012. -P. 224. Germany.ISBN:978-3-659-29945-2 [5] Prokopenya A. N., Minglibayev M. Zh., Kosherbaeva A. B. Derivation of Evolutionary Equations in the Many-Body Problem with Isotropically Varying Masses Using Computer Algebra // Programming and Computer Software. -2022. Vol.48(2). -P. 107-115. DOI:10.1134/S0361768822020098 [6] David Trevor J., Petigura Erik A., Luger Rodrigo, Foreman-Mackey Daniel, Livingston John H., Mamajek Eric E., Hillenbrand Lynne A. Four Newborn Planets Transiting the Young Solar Analog V1298 Tau // The Astrophysical Journal Letters, - 2019, -Vol. 885(1) :L12, 10pp. DOI: 10.3847/2041-8213/ab4c99

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Effects of a finite straight segment on the non-linear stability of the equilibrium point in the planar Robe's problem

The Arnold-Moser theorem (Kolmogorov-Arnold-Moser theory) has been used to study the non-linear stability of the equilibrium point (-\mu, 0 ) in the planar Robe's restricted three-body problem when the second primary is a finite straight segment of length $2 l$. The density parameter $k$ is considered zero. The equilibrium point has been found to be unstable whenever $\backslash m u$ does not belong to the interval $\backslash$ left(8\left(1$\|^{\wedge} 2 \backslash$ right $) / 9,1-I^{\wedge} 2 \backslash$ right $)$. However, in the interval $\backslash \operatorname{left(8\backslash left(1-I^{\wedge }2\backslash right)/9,1-I^{\wedge }2\backslash right),~it~has~been~found~}$ that the Arnold-Moser theorem is not applicable when the mass ratio $\backslash m u$ is equal to one of $\backslash m u \_i, I=1,2,3$, \Idots, 6, where \mu_1 = 0.9371108601 \left(1-|^2\right), \mu_2 = $0.967292242 \backslash$ left(1-|^2\right), \mu_3 = 8/9, and \mu_4, \mu_5, \mu_6 are found graphically. Therefore, no conclusion about the non-linear stability has been drawn for these values of mass ratio $\backslash m u$. It has been found that the equilibrium point is stable in the non-linear sense for all the values of mass ratio $\backslash m u$ in the linear stability interval $\backslash \operatorname{left}(8 \backslash$ left(1$\left.\right|^{\wedge} 2 \backslash$ right $) / 9,1-\left.\right|^{\wedge} 2 \backslash$ right) except $\sim\{\backslash$ mu_i,i=1,2,3, ···, 6$\}$.

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## Effect of straight segment and oblateness on the restricted problem of $2+2$ bodies

In this paper, we investigate the combined effects of the oblateness and straight segment on the positions and linear stability of the equilibrium points in the restricted problem of $2+2$ bodies. The present model holds fourteen equilibrium points, out of which six are collinear with the centers of the primaries and rest are noncollinear. It is observed that the positions of all the equilibrium points are subsequently affected by the oblateness and length of the primary bodies. The linear stability of the equilibrium points is also presented by slightly perturbing the position of the equilibrium points. It is observed that for a considered set of parameters, all the fourteen equilibrium points are unstable. An application of the present model is also

