Phase portraits of the Henon-Heiles potential

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The Henon-Heiles potential is undoubtedly one of the simplest, classical and characteristic examples of open Hamiltonian systems with two degrees of freedom. The above topic was devoted to a large number of research scientists [1,2].

The potential of the Henon-Heiles system is determined by the formula:

$$U(x,y) = \frac{1}{2}(x^2 + y^2 + 2x^2y - \frac{2}{3}y^3) \tag{1}$$

Equation (1) shows that the potential actually consists of two harmonic oscillators, which were connected by the perturbing terms $x^2y - \frac{1}{3}y^3$.

The basic equations of motion for a test particle with a unit mass ($m = 1$) are:

$$\left.\begin{aligned} \ddot{x} &= -\tfrac{\partial U}{\partial x} = -x - 2xy \\ \ddot{y} &= -\tfrac{\partial U}{\partial y} = -y - x^2 + y^2 \end{aligned}\right\} \tag{2}$$

Consequently, the Hamiltonian of system (1) has the form:

$$H = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}(x^2 + y^2) + x^2y - \frac{1}{3}y^3 = h \tag{3}$$

where $\dot{x}$ and $\dot{y}$ are the momenta per unit mass, x and y are the coordinates of the system; $h > 0$ the numerical value of the Hamiltonian, which is conserved. It is seen that $h > 0$ the Hamiltonian is symmetric with respect to $x \to -x$, and H also exhibits a symmetry of rotation at $\frac{2\pi}{3}$.

To study the Henon-Heiles system, the Poincaré section method is used. Advantages of this method are especially evident when we consider nonlinear systems for which exact solutions are unknown. In this case, the phase trajectories are calculated by numerical methods.

To solve the systems of equations (2), boundary conditions are chosen so that they satisfy equation (3). Further, the systems of equation (2) are solved on the basis of the Runge-Kutta method. To construct the Poincaré section, those values that intersect the plane $x = 0$ are chosen. The Poincaré sections for Henon-Heiles systems for different energy values ($E = 1/12$, $E = 1/8$, $E = 1/6$) were investigated. With increasing energy, the structure of the cross sections is destroyed. The results obtained are in agreement with other authors [1, 2].

Thus, the results obtained by the numerical method determine the oscillations for the Henon-Heiles model and serve as the basis for a comparative analysis in determining the analytical mapping.

Keywords: Henon-Heiles model, Poincaré section, numerical solutions.

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References

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